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AN ADAPTIVE LEARNING CONTROL SYSTEM FOR AIRCRAFT

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Dr. Ralph Mekel and Mr. Solomon Nachmias

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1. Introduction

This research is concerned with adaptive learning control systems for aircraft. The research to date, has led to the development of a learning control system which blends the gain scheduling and adaptive control into a single learning system that has the advantages of both. An important feature of the developed learning control system is its capability to adjust the gain schedule in a prescribed manner to account for changing aircraft operating characteristics. Furthermore, if tests performed by the criteria of the learning system preclude any possible change in the gain schedule, then the overall system becomes an ordinary gain scheduling system.

The research accomplished to date is presented in the following sections. First, a statement of the problem is given and then the development of the learning control system is described. Two examples are also discussed. The results of the first example were presented at Langley on July 31, 1975.

2. Statement of Problem

Let the aircraft motion be described by

$$\dot{\underline{x}}_p = A_p(\underline{y}) \cdot \underline{x}_p + B_p(\underline{y}) \cdot \underline{r} \qquad (1)$$

where $\underline{x}_p$ is the state vector and $\underline{r}$ is the input vector. The aircraft is controlled using a feedforward gain matrix G and a feedback gain matrix K. The augmented system is therefore characterized by

$$\dot{\underline{x}}_p = \left[A_p(\underline{y}) + B_p(\underline{y}) \cdot K\right] \underline{x}_p + \left[B_p(\underline{y}) \cdot G\right] \underline{r} \qquad (2)$$

The set of linear equations in Eq. (1) is obtained by linearizing the nonlinear equations of motion in the neighborhood of the operating point $\underline{y}$. After this is done one obtains data points for the elements of the matrices $A_p(\underline{y})$ and $B_p(\underline{y})$ for descrete values of $\underline{y}$. Let $\underline{y} \in Y$, where Y is the vector subspace of admissible operating conditions. Since we are interested in the behavior of the aircraft through the continuum of the subspace Y, we interpolate the data points and therefore the elements of the matrices $A_p(\underline{y})$ and $B_p(\underline{y})$ become functions of the vector $\underline{y}$. Our next step is to obtain a better approximation of the parameters than this initial interpoation has given and also be able to detect changes in the elements of matrices $A_p(\underline{y})$ and $B_p(\underline{y})$ so as to control the aircraft effectively by updating the matrices K and G. For this purpose we developed an adaptive learning control system or briefly a learning control system which is described in the next sections.

3. The Learning Control System

A block diagram illustrating the functional organization of the learning control system is depicted in Fig. 1. One of the features of this system is its capability to adjust the feedforward and feedback gains in a prescribed and learned manner to account for changing aircraft operating characteristics. As shown in Fig. 1., the learning system consists of three basic subsystems: 1. The information acquisition subsystem, 2. The learning algorithm subsystem and 3. The memory and control process subsystem. The tasks of each subsystem and their mathematical development are described in the next three sections.

3.1. The Information Acquisistion Subsystem

The information acquisition subsystem identifies the values of the elements of $A_p(\underline{y})$ and $B_p(\underline{y})$ matrices. Several techniques have been formulated – all were based on the second method of Liapunov to insure convergence of the identification process. The technique that produced the best results is described in this section. In this technique the plant was represented by a model of the form

$$\dot{\underline{x}}_m = F\underline{x}_m + (A_m - F)\underline{x}_p + B_m\underline{r} \tag{3}$$

where F is a stable matrix and the model has the same dimensionality as the plant. The adaptation error is defined as

$$\underline{e} = \underline{x}_m - \underline{x}_p \tag{4}$$

and the error differential equation describing the adaptation error is obtained as

$$\underline{\dot{e}} = F\underline{e} + \left(\sum_{i=1}^{n} \underline{b}_i \underline{u}_i^T\right)\underline{x}_p + \left(\sum_{i=1}^{n} \underline{d}_i \underline{w}_i^T\right)\underline{r} \tag{5}$$

where

$$\sum_{i=1}^{n} \underline{b}_i \underline{u}_i^T = A_m - A_p(\underline{y}) - B_p(\underline{y})\cdot K \tag{6}$$

$$\sum_{i=1}^{n} \underline{d}_i \underline{w}_i^T = B_m - B_p(\underline{y})\cdot G \tag{7}$$

Vectors $\underline{b}_i$ and $\underline{d}_i$ are constant for all i, and $\underline{u}_i$, $\underline{w}_i$ are vectors whose components are the misallignments of the parameters of the i-th row.

An appropriate Liapunov function for Eq. (5) is

$$V = \underline{e}^T M \underline{e} + \sum_{i=1}^{n} \underline{u}_i^T N_i \underline{u}_i + \sum_{i=1}^{n} \underline{w}_i^T Q_i \underline{w}_i \tag{8}$$

where M, N_i and Q_i are symmetric positive definite matrices with constant elements. Applying Liapunov's stability criterion to V and its time derivative $\dot{V}$, one derives a set of controller equations which when related to the model matrices A_m and B_m yields

$$A_m = A_p(\underline{y}) + B_p(\underline{y})\cdot K - \sum_{i=1}^{n} \int_0^t \underline{b}_i \underline{x}_p^T (\underline{b}_i^T M \underline{e}) N_i^{-1} dt \tag{9}$$

$$B_m = B_y(\underline{p})\cdot G - \sum_{i=1}^{n} \int_0^t \underline{d}_i \underline{r}^T (\underline{d}_i^T M \underline{e}) Q_i^{-1} dt \tag{10}$$

The performance of the information acquisition subsystem is monitored by a convergence criterion. This criterion is defined presently by

$$\eta = V(t)/V(0) \leq \eta_{min} \tag{11}$$

where η_{min} is prescribed by the designer according to his desired accuracy of identification for a prescribed length of time while the system is subjected to sufficient excitation. Other performance criteria are also being studied.

3.2. The Learning Algorithm Subsystem

The identified instantaneous values of the parameters affecting the motion of the aircraft are fitted to predetermined analytical expressions describing the behavior of the parameters over the subspace Y. The predetermined analytical expressions were obtained by interpolating the apriori available data and representing it by polynomials. In general, any set of linearly independent functions can be used for this purpose as long as they span the space of the parameter functions.

The elements of the model matrices A_m and B_m are formed into an $n \times 1$ dimensional vector $P_I(\underline{y})$ given by

$$\underline{P}_I(\underline{y}) = H(\underline{y}) \cdot \underline{c} + \underline{v} \tag{12}$$

Vector $\underline{c}$ is to be updated sequentially upon receiving new information ($\underline{P}_I$, $\underline{y}$). This type of learning falls into the category of stochastic approximation method. There are a few ways of updating $\underline{c}$; a survey is given in reference (1). Examining the different algorithms for updating vector $\underline{c}$ one observes that there is a tradeoff between computation complexity and rate of convergence. In our case, we used the least square error algorithm which converges rapidly and is of the form

$$C_{k+1} = C_k + P_k H_{k+1}^T (R_{k+1} + H_{k+1} P_k H_{k+1}^T)^{-1} (P_{I,k+1} - H_{k+1} C_k) \tag{13}$$

$$P_{k+1} = P_k - P_k H_{k+1}^T (R_{k+1} + H_{k+1} P_k H_{k+1}^T)^{-1} H_{k+1} P_k \tag{14}$$

where R_k is the covariance matrix of the error vector

$$v_k = P_{I,k} - H_k C_k \tag{15}$$

For C_0 we use the coefficients of the interpolation polynomials over the apriori available data and P_0 can be chosen any positive definite matrix. The choice of P_0 influences considerably the rate of convergence of the algorithm.

A necessary test for the learning algorithm must be to continually evaluate the validity of the information in vector C_k. This test is performed by the confidence criterion

$$\phi_{k+1} \leq (k+1)\epsilon \tag{16}$$

where $\phi_{k+1} = \sum_{i=1}^{k+1} v_i^T v_i$ or iteratively $\phi_{k+1} = \phi_k + v_{k+1}^T v_{k+1}$.

The quantity ϵ is fixed by the designer, and is the maximum tolerable mean square error.

3.3. The Memory and Control Process Subsystem

After passing the confidence criterion, vector $\underline{C}$ is used to compute the elements of matrices A_m and B_m. The stored values of this vector (in a dynamic memory) are used to compute the elements of matrices $A_p(\underline{y})$ and $B_p(\underline{y})$. The gain matrices K, G are then computed using the equations (6) and (7) by letting $\underline{u}_i = 0$, $\underline{w}_i = 0$. We therefore have

$$K = B_p^{-1}(\underline{y})\left[A_m - A_p(\underline{y})\right] \tag{17}$$

$$G = B_p^{-1}(\underline{y})B_m \tag{18}$$

After the gains are computed, vector $\underline{C}$ updates the previous value in the dynamic memory.

4. Examples

Two examples were studied in order to verify the validity of the learning control system and be able to check out the programs used in the process.

The first example illustrates the learning of three parameter curves for a second order representation of the longitudinal dynamics of an aircraft. The state variables for this system are pitch rate q and pitch attitude θ. The independent variable for the parameter curves is θ. The three parameter curves to be learned are $M_p'(\theta)$, $D_p(\theta)$ and $C_p(\theta)$ -- (moment, damping and control effectiveness). The results were presented at NASA, Langley Research Center on July 31, 1975.

The second example illustrates the learning of 10 parameter curves for a fourth order representation of the longitudinal dynamics of the F-8 aircraft. The state variables of the system are pitch rate q, variational speed V, angle of attack α, and pitch attitude θ. The parameter curves are functions of the mach number M and of the altitude. The equations of motion are linearized at selected flight conditions and can be written as

$$\begin{bmatrix} \dot{q} \\ \dot{V} \\ \dot{\alpha} \\ \dot{\theta} \end{bmatrix} = A_{long} \cdot \begin{bmatrix} q \\ V \\ \alpha \\ \theta \end{bmatrix} + B_{long} \cdot \delta_e \tag{19}$$

where

$$A_{long} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & 0 \\ 0 & a_{22} & a_{23} & -g \\ 1 & a_{32} & a_{33} & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (20)$$

$$B_{long} = \begin{bmatrix} b_{11} \\ b_{12} \\ b_{13} \\ 0 \end{bmatrix} \quad (21)$$

and δ_e is a linear combination of the states and the pilot's input. The a_{ij}'s and b_{ij}'s are given in reference (2). In this example we used data from reference (2) and considered the four wing-down (CO) configurations at sea level; hence the 10 unknown elements of matrices A_{long} and B_{long} were functions of the mach number M. We did not use the data point at M = 1 since it was discontinuous with the rest of the data and it would amount for non valid predetermined functional representations.

Our first step was to interpolate polynomials through the data in the least square sense and determine a functional representation of the system parameters with respect to M. After this was done we had a vector of coefficients ($\underline{C}_m$) for the model. Then we chose a different set of coefficients ($\underline{C}_p$) according to which we computed the plant's unknown parameters for a given mach number. We simulated a flight at sea level at three different mach numbers and the learning system reproduced the parameter curves (a_{ij} & b_{ij} curves).

An interesting observation is that the parameter curves that multiply the state variable V are learned in an easier fashion than the other parameters since the response of V to a given input is sensitive to these parameters. To overcome the difficulty that the state variables q and α are not sensitive to their corresponding parameters we had to give a number of inputs at each operating condition so as to achieve approximate identification and therefore good learning. Note that the a_{ij}'s and b_{ij}'s discussed in this example are related to the stability derivatives for the F-8 aircraft as shown on pg. 3 in reference (2).

5. References

1. Gura, I. A., "An Algebraic Solution of the State Estimation Problem," AIAA Journal, Vol. 7, No. 7, July 1969, pp. 1242-1247.

2. Gera, J., "Linear Equations of Motion for F-8 DFBW Airplane at Selected Flight Conditions," NASA, Langley Research Center, F-8 Digital Fly-By-Wire Internal Document, Report No. 010-74.

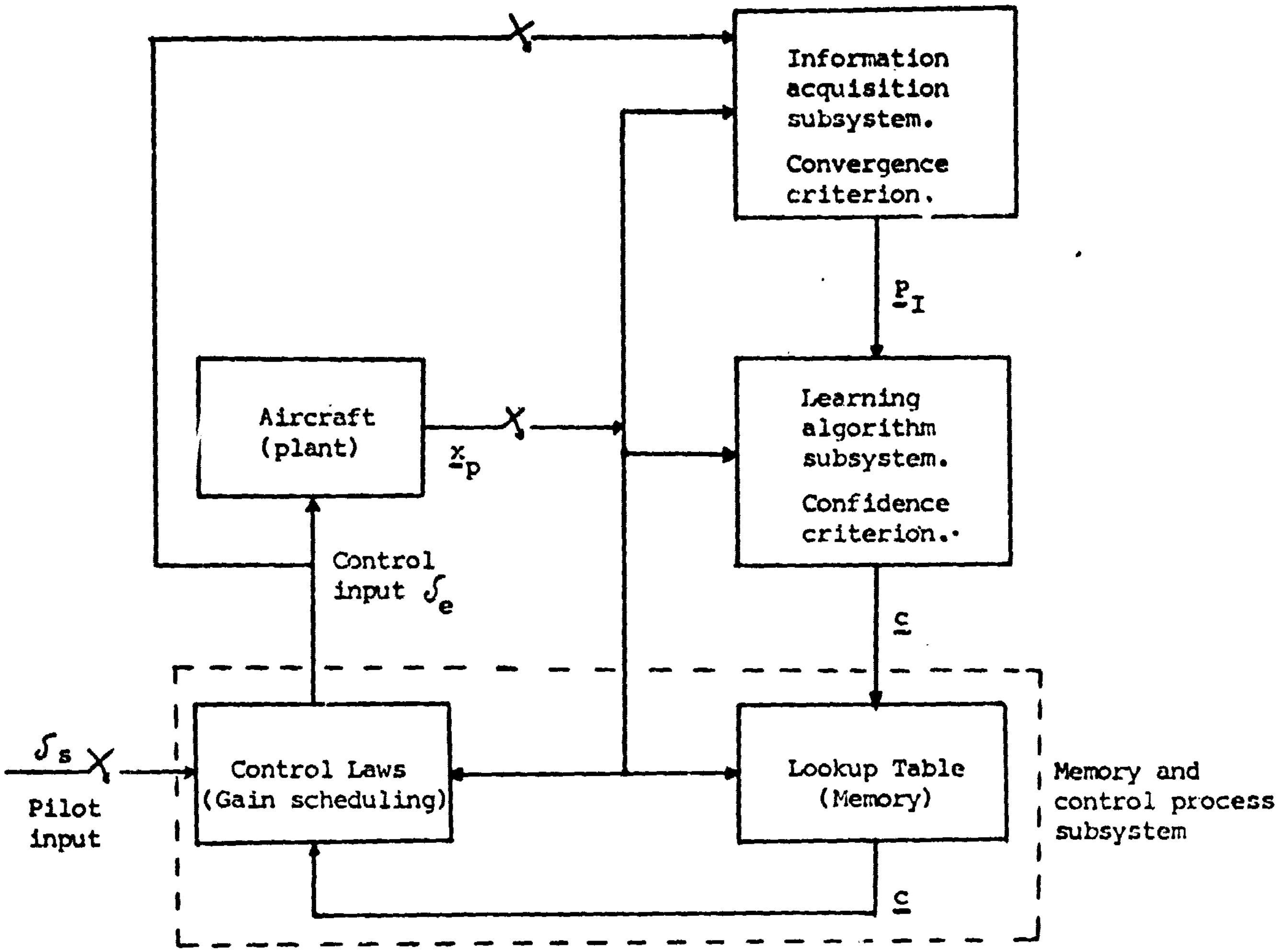

Figure 1. A block diagram illustrating the functional organization of a learning control system.