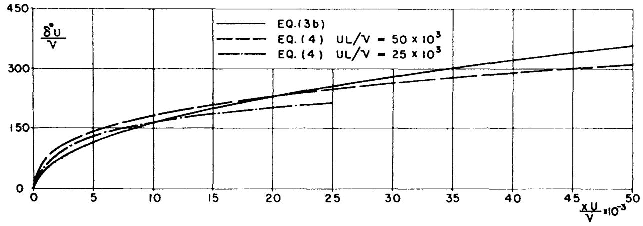

FIG. 1.

therefore the same solution as above, will, of course, result. The method is relatively insensitive to different choices in the assumed function, as may be verified by performing the calculations on the basis of a different function h. For example, if we arbitrarily select

$$h(x) = \alpha x^{1/3} \tag{4}$$

with α the required parameter, then the result is

$$\alpha = (96\nu/7U)^{1/2}L^{1/6} \tag{4a}$$

and, therefore, depends here (as it did not in the previous case) on the length L. The results have meaning only for $x < L$, where they are in good agreement, except near the leading edge, with the ones of Eqs. (3b), (c); this can be seen by the comparison of the boundary-layer thicknesses, obtained by the two methods, plotted in Fig. 1. Of course, for better accuracy for large L, a form for h more general than that of Eq. (4)—i.e., with more parameters, needs to be chosen. The drag coefficients corresponding to the solutions of Eq. (3b), of Eq. (4a), and of Blasius are all of the same form and are, respectively, $1.4/\sqrt{R_L}$, $1.23/\sqrt{R_L}$ and $1.33/\sqrt{R_L}$ where $R_L = UL/\nu$.

In the leading-edge regime, the Prandtl equations are not valid and give rise to singularities in the velocity field; the use of the complete Navier-Stokes equations can be expected[2,3] to eliminate this type of behavior.† We start by assuming that the boundary layer extends a distance a (as yet unknown) ahead of the leading edge along the axis of the plate ($y = 0$). Along $y = 0$, the velocity decreases continuously from the free-stream value U at $x = -a$ to zero at the leading edge $x = 0$. At the same time, the boundary layer grows from zero thickness at $x = -a$ in a manner to be discussed.

A simple flow field which satisfies these conditions is given by the stream function

$$\psi = \begin{cases} U\{[y\,(x+a)^2/2ha^2] + y - [y(x+a)^2/a^2]\}; & -a \le x \le 0,\quad 0 \le y \le h \\ U/2[(y/h)^2] ; & 0 \le x < L,\quad 0 \le y \le h \end{cases} \tag{5}$$

† This problem was considered by Kuo.[4] His solution, obtained by a series approach combined with Lighthill's method for improving higher approximations, shows the great complexity of the problem. Kuo's results are in general agreement with the ones described here.

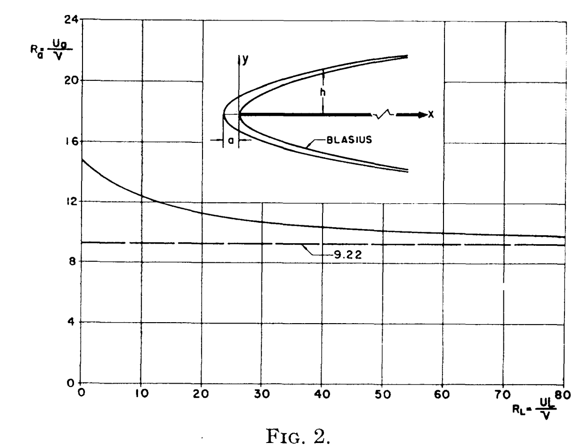

FIG. 2.

where $h(x)$ is the boundary-layer thickness. The unknowns are the parameter a and the function $h(x)$; as a first approximation h was taken to be

$$h(x) = k\sqrt{(x+a)(\nu/U)}; \quad -a \le x$$

namely, that of Eq. (3b) shifted a distance a. The value of k is taken as in Eq. (3c). The only parameter to be determined therefore, is a.

The terms of a stream function, the Navier-Stokes equations require that

$$L(\psi) = \nu\nabla^4\psi - (\partial/\partial y)[(\partial\psi/\partial y)(\partial^2\psi/\partial x\partial y) - (\partial\psi/\partial x)(\partial^2\psi/\partial_y{}^2)] - (\partial/\partial x)[(\partial\psi/\partial x)(\partial^2\psi/\partial x\partial y) - (\partial\psi/\partial y)(\partial^2\psi/\partial_x{}^2)] = 0 \tag{6}$$

This is clearly satisfied if

$$\int_{-a}^{L}\int_{0}^{h(x)} L[\psi]\delta\psi\, dydx = 0 \tag{7}$$

for any arbitrary virtual variation $\delta\psi$. Substitution of Eq. (5) into expression (7) gives upon integration and simplification the following for $R_a = aU/\nu$:

$$c_1R_a{}^2c_2R_a + c_3\{(c_4 + c_5R_L + c_6R_a)/[(R_L/R_a) + 1]^{5/2}\} = 0$$

where

$$R_L = LU/\nu \text{ and } C_1 = 425/1872;\ C_2 = 2399/1055;\ C_3 = 153/400;\ C_4 = 63/80;\ C_5 = 19/48;\ C_6 = 1/3$$

The only admissible root for all choices of R_L is that plotted in Fig. 2, and is virtually independent of the choice of R_L for $R_L > 40$. The boundary-layer height at $x = 0$ is given by $hU/\nu = k\sqrt{R_a}$; away from the leading edge, the boundary layer over the plate very closely resembles that of the classical theory. As a consequence, the drag is almost unaffected by the introduction of the nose region.

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Experimental Verification of Boundary-Layer Corrections in Hypersonic Nozzles

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THE PROBLEM of accurately predicting boundary-layer growth for supersonic nozzle design assumes increased importance as the design Mach Number of the nozzles is pushed into the hypersonic regime. Several methods of calculating this growth have been advanced which vary in ease of computation and accuracy of results. Results of the calibrations performed on two hypersonic helium nozzles recently put into operation in the 11-in. Hypersonic Tunnel Section at the NASA Langley Research Center indicate that the method employed for boundary-layer calculations in these nozzles is adequate. The flow in the test region of both nozzles has been found to be of good quality and the Mach Numbers obtained agree very satisfactorily with the desired design Mach Number.

These nozzles were designed by the method of characteristics[1] to produce uniform parallel flow at Mach Numbers of 10 and 18, the computations of the characteristic net being performed on an IBM 704 calculator. Both nozzles are axisymmetric with a 10.5-in. diameter at the center of the test region. The flow angles at the inflection points of the nozzles were specified as 10° for the Mach-10 nozzle and 12° for the Mach-18 nozzle. The method of computing turbulent boundary-layer displacement effects devised by Persh and Lee[2] was used to obtain the boundary-layer displacement thickness in these nozzles after being adapted for high-speed machine computation. This method, which is based on a finite difference solution of the von Kármán momentum equation, is applicable to either two-dimensional or axisymmetric nozzles and includes the effects of heat transfer. The two problems were set up so that after the ordinates of the flow field were obtained, these results could be used directly as input for the boundary-layer calculations. In order to obtain the desired test section dimensions, several iterations were necessary.

As shown in Fig. 1(a), the Mach-10 nozzle was designed to operate at a stagnation pressure (p_t) of 100 psi. Because of pressure ratio difficulties the nozzle has not been calibrated at this pressure, but an extrapolation of the results obtained at higher pressures indicates that a Mach Number of 10 would be obtained if the nozzle were run at design pressure. The results of the calibration of the Mach-18 nozzle, as shown in Fig. 1(b), indicate that at the design pressure of 1,000 psi the average Mach Number in the center of the test region is 17.8, which differs from design Mach Number by only 1 per cent.

In order to obtain a larger testing area, the length of the Mach-18 nozzle was limited by a method similar to that presented by Kenny and Yu.[3] The agreement between the design longitudinal Mach Number distribution for the Mach-18 nozzle and values obtained experimentally is shown in Fig. 2. The Mach Number gradient along the nozzle axis in the test region is only 0.05 per in. in the Mach-18 nozzle and 0.01 per in. in the Mach-10 nozzle.

The pressure distribution in the test region of both nozzles was determined by a survey with an impact pressure rake. As shown in Fig. 3, the distribution in both nozzles is good, and a moderate change in stagnation pressure effects only a overall shift in the level of the distribution.

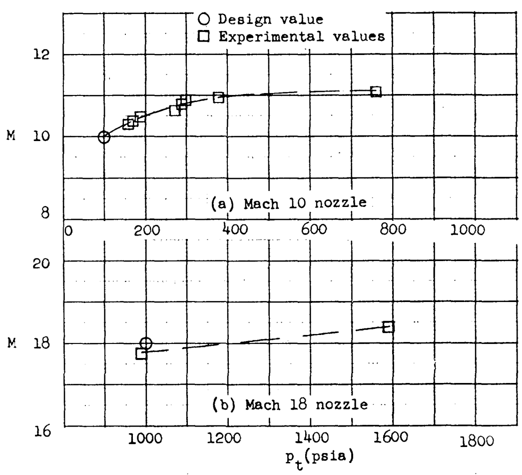

Figure 1. – Comparison of Mach numbers obtained by impact pressure surveys with design Mach numbers for two hypersonic nozzles.

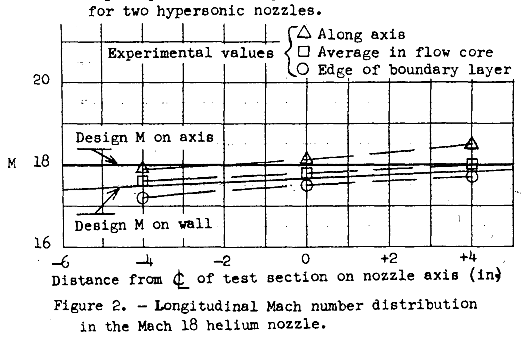

Figure 2. – Longitudinal Mach number distribution in the Mach 18 helium nozzle.

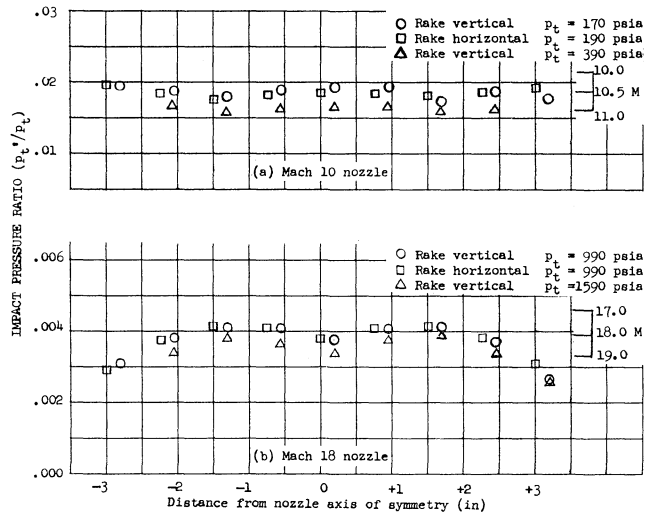

Figure 3. –Results of a survey with an impact pressure rake in the center of the test region in two hypersonic helium nozzles.

Static pressures obtained along the nozzle wall during the calibration runs agree well with the design wall pressures and substantiate the results obtained with the impact pressure rake. The values of Mach Number, M, in the figures were obtained from the impact pressure ratios and are true values in all cases considered here, except those where the boundary layer influences the measured impact pressure. Accordingly, in Fig. 3(b), the Mach Number scale applies only to a region within about 2 in. of the nozzle axis of symmetry.

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Remarks on "The Solution of the Laminar Boundary-Layer Equations"

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EPSTEIN[1] DEVELOPED a new iteration procedure for solving the so-called Prandtl boundary-layer equation. I would like to add a few remarks on this subject and on the subject of the boundary-layer equations in general.

By means of a not one-to-one transformation of coordinates, Prandtl and Blasius passed from a partial differential equation to an ordinary one. All the procedures referring to a solution of the Prandtl-Blasius equation (Epstein's as well) refer to an ordinary differential equation. Due to the existence of complex boundary conditions and the lack of a one-to-one transformation, it is not possible—using the available methods of the theory of functions—to prove anything back in the domain of the partial differential equation. This can be done by means of some algebraic methods (Michal,[2] Morgan,[3] *et al.*) but only for an equation without any boundary conditions. Thus, an exact solution in the domain of an ordinary differential equation does not need to be necessarily an exact solution of the original partial differential equation. As a matter of fact, absolutely nothing can be said on that at the present time. There are some voices here and there, that the