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EFFECTS OF GYROSCOPIC CROSS COUPLING BETWEEN PITCH AND ROLL ON THE HANDLING QUALITIES OF VTOL AIRCRAFT

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SUMMARY

In order to provide information relative to the effects of gyroscopic cross coupling between pitch and roll on the handling qualities of VTOL aircraft, a flight investigation has been conducted during which cross coupling was simulated. Generality is achieved by presenting the results of the flight investigation in the form of a criterion which may be used to predict the acceptability of the level of cross coupling in VTOL aircraft as a function of the aircraft design parameters. The criterion is based on pilots' opinions of the acceptability of the aircraft motions for the range of cross coupling which was simulated during a maneuver in which cross coupling is particularly objectionable. Theory is used to provide a basis for application of the criterion. The theory which is developed is shown to predict accurately the aircraft motions.

The pilots agreed that the available control power determines to a great extent the amount of cross coupling which can be tolerated. The theoretical investigation indicates the extent to which the damping and moments of inertia of the aircraft as well as the angular momentum of the engine influence cross coupling.

INTRODUCTION

Recent jet VTOL aircraft have exhibited undesirable motions attributable to gyroscopic moments arising from the large angular momentum produced by the jet engines. As stated in reference 1, even larger gyroscopic moments than those present in conventional jet aircraft are likely to be present in VTOL aircraft. This situation arises because VTOL aircraft generally require a greater thrust weight ratio than conventional aircraft; therefore, other factors being equal, the engine dimensions and weight are increased, and hence angular momentum is increased. Furthermore, since the provision of high control power and high damping in VTOL configurations is likely to involve special penalties, the VTOL aircraft may tend to have low values of control power and damping as compared with high-speed airplanes. Thus, even if the angular momentum of engines for VTOL aircraft were not greater, the relatively poor control power and low damping in the low-speed and hovering ranges would tend to highlight the gyroscopic effects for these flight conditions.

Coupling of aircraft motions due to gyroscopic moments occurs whenever an aircraft contains rotating components having a net angular momentum in any direction. If it is assumed that the angular momentum vector (as determined by the right-hand rule) is along one of the three principal inertia axes, cross coupling will be present between the other two axes. This cross coupling is characterized by a gyroscopic moment about one axis proportional to the angular velocity about the other axis. Specifically, in the case of pitch-roll cross coupling, a pitching acceleration proportional to the rolling velocity is produced and a rolling acceleration proportional to the pitching velocity is produced.

The source of gyroscopic cross coupling between pitch and roll is mass rotating about the vertical axis - for example, jet engines mounted vertically. Although cross coupling may be eliminated by the use of counterrotating engines or by electronics, the aircraft must be controllable in the event of failure of one or more of several counterrotating engines or of the electronic system.

A variable-stability research helicopter was used to simulate a wide range of cross coupling. An investigation of the angular momentum of typical present-day jet engines and of the probable size of aircraft in which these engines may be utilized indicated that the range of coupled responses simulated was adequate. The flight tests included several different flight conditions selected as representative of maneuvers normally associated with VTOL aircraft, including maneuvers in which cross coupling is expected to present the greatest problems. The results of these tests, along with a discussion of the effects on the coupled response (response produced by gyroscopic moment) of various parameters such as the damping and moment of inertia of the aircraft, are presented herein. Generality is achieved by presenting the results of the flight investigation in the form of a criterion which may be used to predict the acceptability of the level of cross coupling as a function of the aircraft design parameters. The criterion is based on pilots' opinions of the acceptability of the aircraft motions for the range of cross coupling which was simulated during a maneuver in which cross coupling is particularly objectionable. Theory is used to provide a basis for the application of the criterion. The theory which is developed is shown to predict accurately the aircraft motions.

Inasmuch as initial tests indicated that the gyroscopic coupling never produced an objectionable response about the longitudinal axis, this investigation deals primarily with the coupled response occurring about the lateral axis, that is, the pitching acceleration proportional to rolling velocity.

SYMBOLS

M_p	damping moment proportional to and opposing rolling velocity, lb-ft/radian/sec
M_q	damping moment proportional to and opposing pitching velocity, lb-ft/radian/sec
H	net angular momentum of mass rotating about vertical axis, slug-ft^2/sec
I_X	moment of inertia about body X-axis, slug-ft^2
I_Y	moment of inertia about body Y-axis, slug-ft^2
I_Z	moment of inertia about body Z-axis, slug-ft^2
M_δ	lateral control moment per inch stick deflection, lb-ft/in.
p	rolling velocity, radians/sec
$\dot{p}$	rolling accelerations, radians/sec^2
$\bar{p}$	Laplace transform of p
q	pitching velocity, radians/sec
$\dot{q}$	pitching acceleration, radians/sec^2
$\bar{q}$	Laplace transform of q
s	Laplacian variable
t	time, sec
δ	stick displacement, in.

TEST EQUIPMENT AND PROCEDURES

Helicopter

All flights in which gyroscopic cross coupling between pitch and roll motions was simulated were performed with the variable-stability helicopter shown in figure 1. The test helicopter contains a variable control system which makes it possible to vary both the ratio of control moment to stick deflection, or control power, and the apparent angular-velocity damping about each of the three principal inertia axes. The variable control system, components of which were used to produce the cross coupling, is described in reference 2. The helicopter is equipped to record angular velocity about all three axes, airspeed, and all control motions of the pilot. The general physical characteristics of the helicopter are given in table I.

Simulation

The test helicopter was the simulator. For the studies of reference 2, signals proportional to helicopter rate operated continuously about all three axes to provide additional angular-velocity damping. These signals were generated by rate gyroscopes. For purposes of this investigation, the simulation was accomplished by repositioning both the pitch-rate gyroscope and the roll-rate gyroscope so that a signal proportional to the rolling velocity would produce a pitching acceleration and a signal proportional to the pitching velocity would produce a rolling acceleration.

Flight Conditions

The effects of gyroscopic cross coupling on the handling qualities of the aircraft were investigated during hovering, low-speed instrument-landing-system (ILS) approaches, steep turns, and rapid roll reversals at 30 to 35 knots. The range of cross coupling covered during each flight condition, in terms of its most significant parameter - the ratio of angular momentum to moment of inertia, is listed in table II. For an aircraft with negligible damping about its coupled axis (axis about which coupled response is produced), this ratio is numerically equal to the acceleration produced about the coupled axis per unit angular velocity about the other axis.

THEORY

The magnitude of the motions due to gyroscopic cross coupling simulated in the test helicopter was predicted by using the equations developed in the appendix. Equations (5) or (6) of the appendix express the rolling and pitching angular-velocity responses resulting from a lateral step input as functions of the moments of inertia and damping about each of the coupled axes, the control power about the lateral axis, and the net angular momentum of the rotating components. Specifically, equation (6a) gives the lateral angular-velocity response to a lateral step input and equation (6b) gives the simultaneous longitudinal angular-velocity response - the coupled response. Thus, with the use of these equations, angular-velocity time histories may be obtained for any aircraft for which the cited parameters are available. By comparison of these computed time histories with time histories which have been correlated with aircraft motions which were, in turn, correlated with pilots' opinions from flight tests, a qualitative estimate can be made of the acceptability of the cross coupling present.

RESULTS

Flight Results

Inasmuch as initial flight tests indicated that a rolling response due to pitching velocity was not a problem in any of the maneuvers, attention was focused primarily on pitching response due to rolling velocity.

Hovering.- During the hovering maneuver, which involved only attempts at holding the aircraft absolutely motionless, the coupled response (pitch acceleration due to roll velocity) about the longitudinal axis was varied over the range given in table II. Even at the maximum value of coupling, the pilot reported that no coupled response was apparent.

Low-speed ILS approaches.- Low-speed ILS approaches were made by utilizing several longitudinal control powers, each with various amounts of cross coupling within the range indicated in table II. The coupled response became objectionable only for the condition in which the cross coupling had the maximum value given for this maneuver and the longitudinal control power was simultaneously one-half that of the basic test helicopter. For this extreme condition the pilot stated that occasionally he was forced to use maximum available longitudinal control to correct for the coupled response.

Steep turns and roll reversals.- Steep turns and roll reversals are quite similar to one another from the standpoint of cross-coupling effects. The latter maneuver (roll reversals) was selected as the focal point of this study because it permitted greater roll rates of longer duration than were feasible or even possible with any of the other maneuvers. Average roll rates of 0.5 radian/sec were maintained during attitude changes from bank angles of 30° right to 30° left while the cross coupling was varied over the range indicated in table II. For this maneuver, ratings of various amounts of gyroscopic cross coupling were obtained from three pilots. The pilots' ratings for the different values of cross coupling were in substantial agreement. The results are tabulated as follows:

Gyroscopic cross coupling, $\frac{H}{I_Y}$, $\frac{\text{radians/sec}^2}{\text{radians/sec}}$	Pilots' rating
0.11	Acceptable
.22	Marginal
.33	Poor
.44	Unacceptable

Analytical Results

The aircraft motions for a lateral step input were correlated with computed angular-velocity time histories predicted by equations (6) in the appendix. Figure 2 contains a plot of equations (6) for the known helicopter parameters and a selected value of angular momentum H. The test-point symbols, which indicate reasonably good agreement with the theory, represent the experimental angular-velocity response about the roll axis and the pitch axis for a lateral step input for the same value of angular momentum H set into the simulator.

Although the pilots' ratings of various amounts of cross coupling for a sustained roll-reversal maneuver have been tabulated in terms of H/I_Y, in essence, however, the pilot is actually rating the amount of coupled response $\dot{q}/p$ which he experiences for a given lateral input. The ratio $\dot{q}/p$ is a function both of H/I_Y and M_q/I_Y. For the roll-reversal maneuver the input resulted in an average roll rate of about 0.5 radian/sec; the coupled response was the pitching velocity, which the pilot attempted to nullify with a marginal longitudinal control system. On the basis of the pilots' ratings for the different amounts of gyroscopic coupling and the known physical characteristics of the test helicopter, the curves in figure 3 were plotted by using equations (5) in the appendix. The dashed curve represents the lateral angular velocity resulting from the lateral input. The two solid curves, which form the cross-coupling boundaries, represent the coupled response about the pitch axis for different values of gyroscopic coupling but for the same lateral input.

DISCUSSION

The relative insignificance of gyroscopic effects during the precision-type maneuvers of hovering and low-speed ILS approaches is attributed to the absence of both large pitching and large rolling motions of the aircraft. For a similar reason, that is, absence of large pitching velocities in all maneuvers, the pilots reported that there was little or no apparent rolling due to cross coupling during any of the maneuvers. Only pitching due to roll was of sufficient magnitude to cause a problem.

Since it was not possible to vary the aircraft damping about the coupled axes during the simulation (the variable damping system was being used to provide the coupling signal), the effect of angular-velocity damping on the coupled response was calculated by using the equations presented in the appendix. Figure 4 contrasts the coupled response - longitudinal response for a lateral input - of an aircraft with zero longitudinal damping and the coupled response of the same aircraft with its longitudinal damping increased to that found desirable for hovering and low-speed flight in reference 3. A more quantitative measure of the effect of damping may be obtained by comparing the steady-state angular velocity about the input axis with that about the coupled axis for a step input. With the use of equations (5) or (6) in the appendix, the ratio of steady-state angular velocities $(q/p)_{t=\infty}$ is given by H/M_q. Hence, for a given input angular velocity about the lateral axis, the steady-state angular velocity about the longitudinal axis is inversely proportional to the longitudinal damping.

In light of the foregoing statements, the ratio of net angular momentum to aircraft moment of inertia is not a reliable criterion for determining the magnitude and acceptability of the cross coupling present in a given aircraft, since it does not take into account the aircraft damping. Therefore, for an aircraft in which cross coupling is anticipated, it is advisable to compare computed angular-velocity time histories with those given in figure 3 in order to determine the acceptability of cross coupling. However, it should be noted that the ratings presented in figure 3 were established for a specific maneuver and for the basic control power of the test vehicle. For an aircraft performing maneuvers involving roll rates greater than 0.5 radian/sec the requirements would be more stringent. The pilots agreed that with greater available control power they would be able to tolerate more cross coupling. Conversely, with less available control power, the amount of cross coupling which could be tolerated would be correspondingly decreased.

The fact that recent VTOL aircraft have exhibited adverse cross-coupling effects while performing even mild maneuvers such as precision hovering is attributed in part to their negligibly low damping and to their low control power during this flight condition. The low control power and low damping, which are present in many of the past VTOL aircraft designs during hovering and transition phases, are traceable to a lack of a ready source for producing desirable control and damping moment during this flight condition. Although increases in both control power and damping are expected, the increases are not likely to eliminate entirely the problems associated with cross coupling for all maneuvers. Thus a means should be sought to minimize the gyroscopic moment in its own right.

CONCLUSIONS

As a result of flight tests and a theoretical investigation of gyroscopic cross coupling between pitch and roll in VTOL aircraft, the following conclusions are drawn:

1. For aircraft which meet requirements for control power and angular-velocity damping, gyroscopic cross coupling between pitch and roll is not likely to present a problem for mild, precision maneuvers such as low-speed instrument-landing-system approaches or hovering. The roll rates encountered during these maneuvers might be similar to those encountered in commerical-type operations such as transport or passenger carrying.

2. Cross coupling is a problem in sustained roll maneuvers performed at low speed such as might be encountered in military operations, for example, where the aircraft is used as a weapon platform and considerable maneuvering might be required for moving into and away from the combat area.

3. A coupled response is more likely to occur and to be a problem about the pitch axis rather than about the roll axis, because of the absence of large pitching velocities in the majority of maneuvers.

4. The magnitude of the coupled response which will occur in a particular aircraft is inversely proportional to the aircraft moment of inertia about the coupled axis and decreases significantly with increased damping moment about the coupled axis. The amount of coupled response

which can be tolerated is increased with an increase in longitudinal control power inasmuch as it enables the pilot to compensate more readily for the coupling.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., February 13, 1961.

APPENDIX

DERIVATION OF ANGULAR-VELOCITY RESPONSE FOR A LATERAL STEP INPUT WITH CROSS COUPLING PRESENT

For a lateral displace-and-hold type of input, the moment equations about the lateral and longitudinal axes, respectively, are

$$\left.\begin{aligned} \dot{p} + \frac{M_p}{I_X} p + \frac{H}{I_X} q &= \delta \frac{M_\delta}{I_X} \\ \dot{q} + \frac{M_q}{I_Y} q - \frac{H}{I_Y} p &= 0 \end{aligned}\right\} \tag{1}$$

Assuming the initial conditions $p(0) = q(0) = 0$ and taking the Laplace transform of equations (1) yields

$$\left.\begin{aligned} s\bar{p} + \frac{M_p}{I_X} \bar{p} + \frac{H}{I_X} \bar{q} &= \delta \frac{M_\delta}{I_X} \frac{1}{s} \\ s\bar{q} + \frac{M_q}{I_Y} \bar{q} - \frac{H}{I_Y} \bar{p} &= 0 \end{aligned}\right\} \tag{2}$$

Solving algebraically for $\bar{p}$ and $\bar{q}$ gives

$$\bar{p} = \delta \frac{M_\delta}{I_X} \frac{1}{\left[s + \dfrac{\dfrac{M_p}{I_X} + \dfrac{M_q}{I_Y} + \sqrt{\left(\dfrac{M_p}{I_X} - \dfrac{M_q}{I_Y}\right)^2 - \dfrac{4H^2}{I_X I_Y}}}{2}\right]\left[s + \dfrac{\dfrac{M_p}{I_X} + \dfrac{M_q}{I_Y} - \sqrt{\left(\dfrac{M_p}{I_X} - \dfrac{M_q}{I_Y}\right)^2 - \dfrac{4H^2}{I_X I_Y}}}{2}\right]}$$

$$+ \delta \frac{M_\delta}{I_X} \frac{M_q}{I_Y} \frac{1}{s\left[s + \dfrac{\dfrac{M_p}{I_X} + \dfrac{M_q}{I_Y} + \sqrt{\left(\dfrac{M_p}{I_X} - \dfrac{M_q}{I_Y}\right)^2 - \dfrac{4H^2}{I_X I_Y}}}{2}\right]\left[s + \dfrac{\dfrac{M_p}{I_X} + \dfrac{M_q}{I_Y} - \sqrt{\left(\dfrac{M_p}{I_X} - \dfrac{M_q}{I_Y}\right)^2 - \dfrac{4H^2}{I_X I_Y}}}{2}\right]} \tag{3a}$$

$$\bar{q} = -\delta \frac{M_\delta}{I_X} \frac{H}{I_Y} \frac{1}{s\left[s + \dfrac{\dfrac{M_p}{I_X} + \dfrac{M_q}{I_Y} + \sqrt{\left(\dfrac{M_p}{I_X} - \dfrac{M_q}{I_Y}\right)^2 - \dfrac{4H^2}{I_X I_Y}}}{2}\right]\left[s + \dfrac{\dfrac{M_p}{I_X} + \dfrac{M_q}{I_Y} - \sqrt{\left(\dfrac{M_p}{I_X} - \dfrac{M_q}{I_Y}\right)^2 - \dfrac{4H^2}{I_X I_Y}}}{2}\right]} \tag{3b}$$

After the inverse transform is taken and the terms are rearranged, equations (3) become

$$p = \delta \frac{M_\delta}{I_X}\left[-\frac{\frac{M_p}{I_X} - \frac{M_q}{I_Y} + \sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}}{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y} + \left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)\sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}}\right] \exp\left[-\frac{\frac{M_p}{I_X} + \frac{M_q}{I_Y} + \sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}}{2}\, t\right]$$

$$- \delta \frac{M_\delta}{I_X} \frac{\frac{M_p}{I_X} - \frac{M_q}{I_Y} - \sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}}{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y} - \left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)\sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}} \exp\left[-\frac{\frac{M_p}{I_X} + \frac{M_q}{I_Y} - \sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}}{2}\, t\right]$$

$$+ \delta \frac{M_\delta}{I_X} \frac{M_q}{I_Y} \frac{1}{\frac{M_p M_q}{I_X I_Y} + \frac{H^2}{I_X I_Y}} \tag{4a}$$

$$q = \delta \frac{M_\delta}{I_X} \frac{H}{I_Y} \frac{2}{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y} + \left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)\sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}} \exp\left[-\frac{\frac{M_p}{I_X} + \frac{M_q}{I_Y} + \sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}}{2}\, t\right]$$

$$+ \delta \frac{M_\delta}{I_X} \frac{H}{I_Y} \frac{2}{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y} - \left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)\sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}} \exp\left[-\frac{\frac{M_p}{I_X} + \frac{M_q}{I_Y} - \sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}}{2}\, t\right]$$

$$+ \delta \frac{M_\delta}{I_X} \frac{H}{I_Y} \frac{1}{\frac{M_p}{I_X} \frac{M_q}{I_Y} + \frac{H^2}{I_X I_Y}} \tag{4b}$$

Equations (4), when simplified, become

$$p = -\delta \frac{M_\delta}{I_X} \frac{1}{\frac{M_p}{I_X}\frac{M_q}{I_Y} + \frac{H^2}{I_X I_Y}} \left\{ \frac{M_q}{I_Y} \cosh\left[\frac{t}{2}\sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}\right] \exp\left[-\frac{1}{2}\left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)t\right] \right.$$

$$\left. - \frac{\frac{M_p}{I_X}\frac{M_q}{I_Y} - \left(\frac{M_q}{I_Y}\right)^2 + \frac{2H^2}{I_X I_Y}}{\sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}} \sinh\left[\frac{t}{2}\sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}\right] \exp\left[-\frac{1}{2}\left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)t\right] - \frac{M_q}{I_Y} \right\} \tag{5a}$$

$$q = -\delta \frac{M_\delta}{I_X} \frac{H}{I_Y} \frac{1}{\frac{M_p}{I_X}\frac{M_q}{I_Y} + \frac{H^2}{I_X I_Y}} \left\{ \cosh\left[\frac{t}{2}\sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}\right] \exp\left[-\frac{1}{2}\left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)t\right] \right.$$

$$\left. + \frac{\frac{M_p}{I_X} + \frac{M_q}{I_Y}}{\sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}} \sinh\left[\frac{t}{2}\sqrt{\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y}}\right] \exp\left[-\frac{1}{2}\left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)t\right] - 1 \right\} \tag{5b}$$

The lateral and longitudinal angular-velocity responses to a lateral step input are given by equations (5) when $\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y} \gtreqless 0$. When $\left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2 - \frac{4H^2}{I_X I_Y} < 0$, equations (5) become

$$p = -\delta \frac{M_\delta}{I_X} \frac{1}{\frac{M_p}{I_X}\frac{M_q}{I_Y} + \frac{H^2}{I_X I_Y}} \left\{ \frac{M_q}{I_Y} \cos\left[\frac{t}{2}\sqrt{\frac{4H^2}{I_X I_Y} - \left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2}\right] \exp\left[-\frac{1}{2}\left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)t\right] \right.$$

$$\left. - \frac{\frac{M_p}{I_X}\frac{M_q}{I_Y} - \left(\frac{M_q}{I_Y}\right)^2 + \frac{2H^2}{I_X I_Y}}{\sqrt{\frac{4H^2}{I_X I_Y} - \left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2}} \sin\left[\frac{t}{2}\sqrt{\frac{4H^2}{I_X I_Y} - \left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2}\right] \exp\left[-\frac{1}{2}\left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)t\right] - \frac{M_q}{I_Y} \right\} \tag{6a}$$

$$q = -\delta \frac{M_\delta}{I_X} \frac{H}{I_Y} \frac{1}{\frac{M_p}{I_X}\frac{M_q}{I_Y} + \frac{H^2}{I_X I_Y}} \left\{ \cos\left[\frac{t}{2}\sqrt{\frac{4H^2}{I_X I_Y} - \left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2}\right] \exp\left[-\frac{1}{2}\left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)t\right] \right.$$

$$\left. + \frac{\frac{M_p}{I_X} + \frac{M_q}{I_Y}}{\sqrt{\frac{4H^2}{I_X I_Y} - \left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2}} \sin\left[\frac{t}{2}\sqrt{\frac{4H^2}{I_X I_Y} - \left(\frac{M_p}{I_X} - \frac{M_q}{I_Y}\right)^2}\right] \exp\left[-\frac{1}{2}\left(\frac{M_p}{I_X} + \frac{M_q}{I_Y}\right)t\right] - 1 \right\} \tag{6b}$$

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3. Salmirs, Seymour, and Tapscott, Robert J.: The Effects of Various Combinations of Damping and Control Power on Helicopter Handling Qualities During Both Instrument and Visual Flight. NASA TN D-58, 1959.

TABLE I.- PHYSICAL CHARACTERISTICS OF THE TEST HELICOPTER

Gross weight, lb	5,500
Moments of inertia:	
Pitch, I_Y, slug-ft^2	7,000
Roll, I_X, slug-ft^2	2,000
Yaw, I_Z, slug-ft^2	5,000
Number of blades in main rotor	3
Rotor rotational speed, radians/sec	19.4
Rotor diameter, ft	48
Height of rotor hub with respect to center of gravity, ft	6.5
Blade mass factor	9
Control travel:	
Longitudinal cyclic, in.	13.6
Lateral cyclic, in.	13.6
Pedal, in.	4.75
Basic control power:	
Pitch, ft-lb/in. of control travel	508
Roll, ft-lb/in. of control travel	474
Yaw, ft-lb/in. of control travel	4,140
Basic damping:	
Pitch, ft-lb/radian/sec	2,495
Roll, ft-lb/radian/sec	2,495
Yaw, ft-lb/radian/sec	10,600

TABLE II.- RANGE OF CROSS COUPLING SIMULATED

Flight condition	Pitch, $\frac{H}{I_Y}$, $\frac{\text{radians/sec}^2}{\text{radians/sec}}$	Roll, $\frac{H}{I_X}$, $\frac{\text{radians/sec}^2}{\text{radians/sec}}$
Hovering	0 to 1.40	0 to 1.05
Low-speed ILS approaches	0 to .60	0 to 1.05
Steep turns	0 to .44	0 to 1.05
Roll reversals	0 to .44	0 to 1.05

Figure 1.- Helicopter used in investigation. L-88302.1

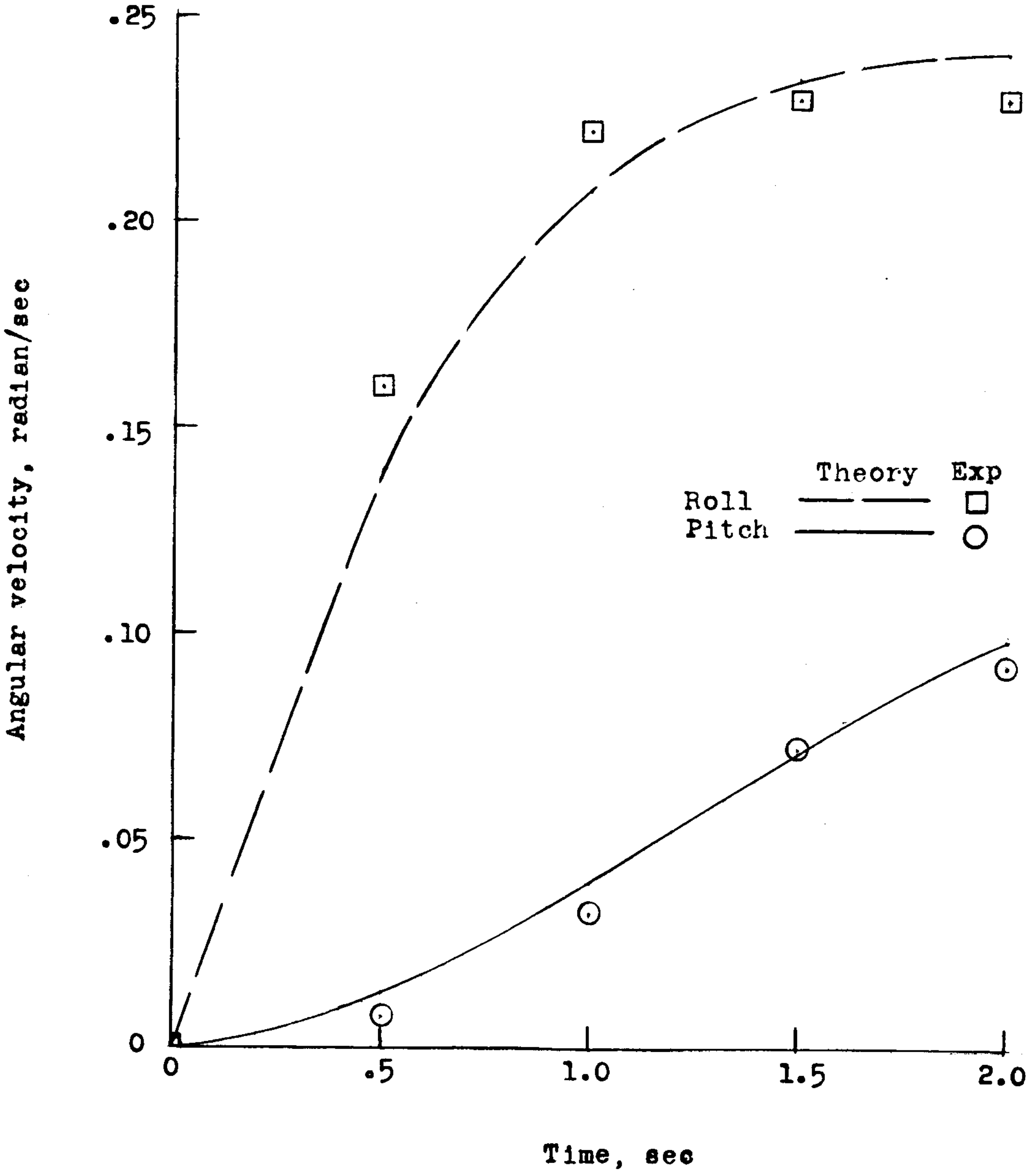

Figure 2.- Comparison of predicted and experimental angular-velocity time histories for a lateral step input.

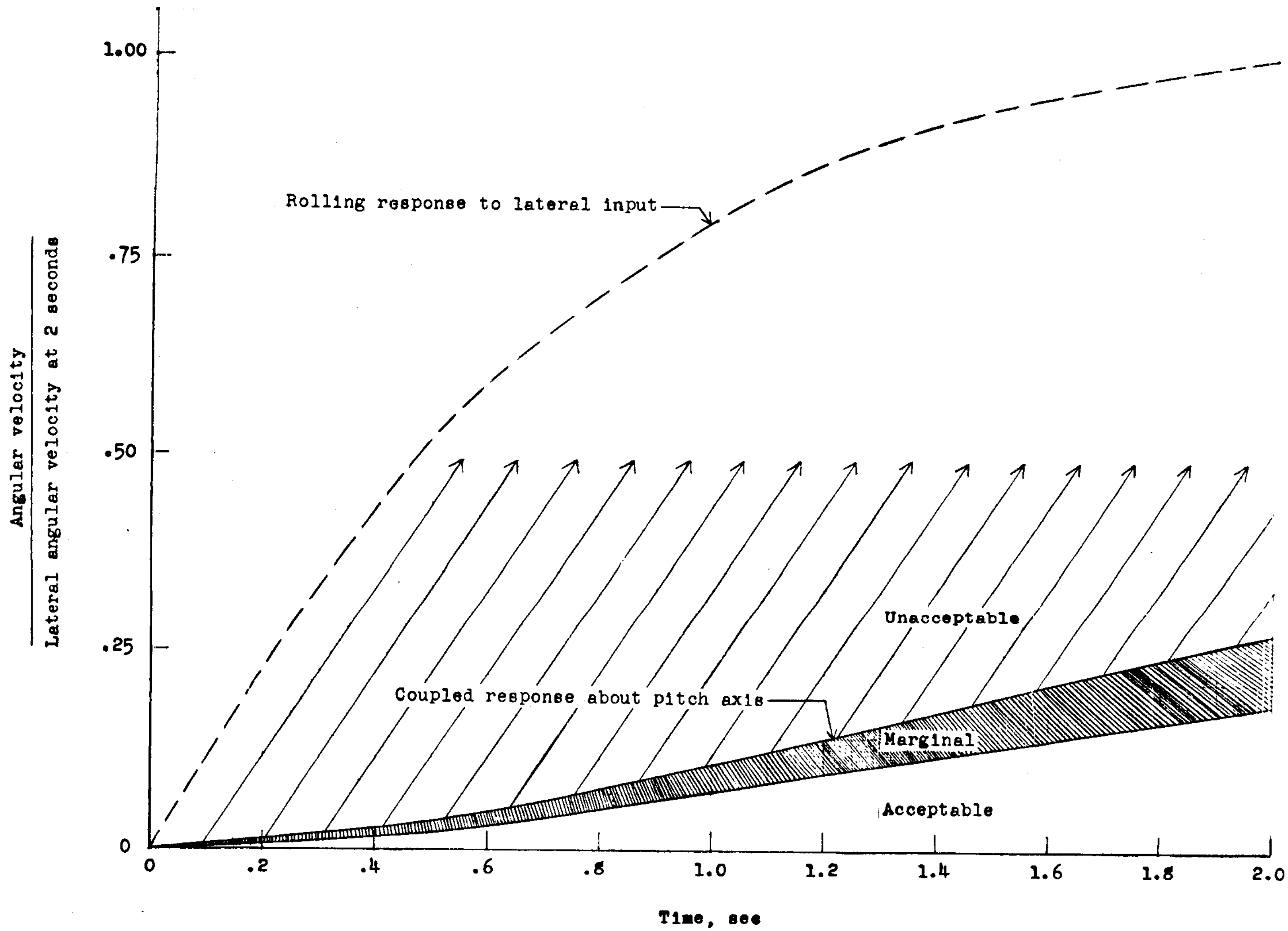

Figure 3.- Cross-coupling boundaries for sustained rolling maneuvers for basic available control power of test helicopter.

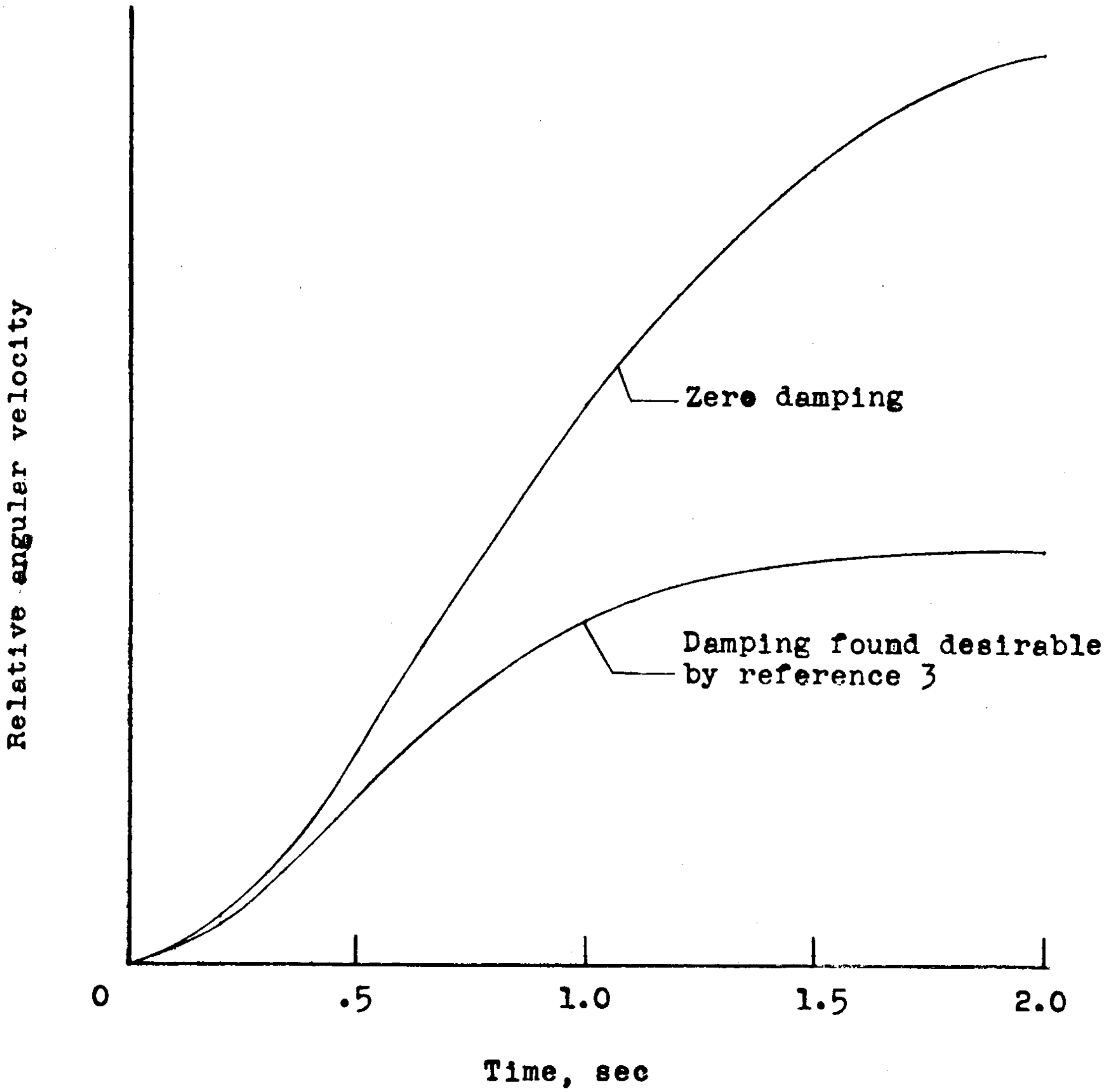

Figure 4.- The effect of longitudinal damping on the longitudinal response for a lateral input.