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DECISION AND CONTROL SCIENCES GROUP
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STATUS REPORT ON
CONTROL OPTIMIZATION, STABILIZATION
AND COMPUTER ALGORITHMS FOR
AIRCRAFT APPLICATIONS

Report ESL-SR-634
M.I.T. Project OSP 76265
Eighteenth Status Report

TO:

1) Office of Research Grants and Contracts (Code SC)
   National Aeronautics and Space Administration
   Washington, D.C. 20546

2) NASA Scientific and Technical Information Facility
   P.O. Box 33
   College Park, Maryland 20740

3) Mr. Brian F. Doolin
   Leader, Control Theory and Applications Group
   FSC: 210-3
   NASA/AMES Research Center
   Moffett Field, California 94035

SUBJECT: Research Grant No. NGL-22-009-124
             Control Optimization, Stabilization and Computer
               Algorithms for Aircraft Applications
             Eighteenth Status Report for Period 1 June 1975
             to 1 December 1975.

DATE: December 2, 1975

A. OVERVIEW

Our research during this time period January 1 to June 1, 1975 will be summarized under the following topics (see Section D)

1) Systems Reliability Optimization

2) Failure Detection Algorithms

3) Analysis of Nonlinear Filters

4) Design of Compensators Incorporating Time Delays

5) Digital Compensator Design

6) Estimation for Systems with Echoes

7) Low-Order Compensator Design

8) Descent-Phase Controller for 4-D Navigation

9) Infinite Dimensional Mathematical Programming Problems and Optimal Control Problems with Constraints

10) Robust Compensator Design

11) Numerical Methods for the Lyapunov Equations

12) Perturbation Methods in Linear Filtering and Control.

B. PERSONNEL

During this time period the following received partial salary support under this grant

Faculty

Professor M. Athans

Professor T.L. Johnson

Professor S.K. Mitter

Professor P. Varaiya

Professor A.S. Willsky

Research Assistants

Mr. B. Llorens   Mr. M. Safanov

Mr. R. Kwong   Mr. T. Athay

Mr. P.K. Wong   Mr. D. Teneketzis

In addition the following contributed to the overall research effort but received no direct salary support during this time period.

Faculty and Research Staff

Prof. N.R. Sandell

Dr. S.B. Gershwin

Dr. K.-P. Dunn

Students

A. Motazedi   R. Lee   K. Tong

F. Lax   R. Ku

S. Marcus   E. Chow

R. Bueno   V. Klebanoff

G. Roberts   J. Eterno

C. INTERACTIONS

The major interaction during this time period between M.I.T. personnel and NASA/Ames research staff occurred during the M.I.T.- NASA Ames Workshop on System Reliability Issues for Future Aircraft. This meeting, held at M.I.T. from August 17-20, brought together a number of prominent researchers, including many from NASA/Ames and M.I.T., to discuss a number of aspects of their work that impact on the question of reliable aircraft design. Some of the work outlined in Section D -- specifically that related to reliability and failure detection -- was reported at that meeting.

D. Technical Discussion

1. Systems Reliability Optimization

To date, research by Mr. Douglas Birdwell and Professor Mike Athans has been concerned with the representation of systems for use in the study of systems reliability and optimization. A theoretical framework is sought which is general enough to consider many different types of problems, yet is specific enough to allow optimal design of reliable systems. The framework currently being investigated accounts for all possible discrete failures of a system and allows for optimization over a set of system configurations which meet a specified performance criterion. Given a specific problem, this approach seems to work well and has given good results for the problem of sensor allocation for a linear system, where failure constitutes the loss of some set of sensors. The approach also seems to be applicable to other areas, such as the choice of linearizations for use in the MMAC algorithms for observation and control of significantly nonlinear systems. Some problems with the approach include the theoretical and computational complexity encountered in the application of the method. The theoretical complexity may be reduced in the future with the addition of general results applicable to large sets of problems. Ways may be found to reduce the computational complexity through the careful use of approximations involving the set containing all possible modes of failure.

A branch of research which has been induced by the above work involves a study of multiple model adaptive control of a nonlinear system. A new approach has been outlined which considers the set of linearizations of the system function to be a finite-state space. The problem being

investigated is as follows: If a control system using some form of MMAC

is approximating the system with some subset of available linearizations

S at some time t, how should the control law choose what subset of

linearizations S' to use at some future time t'? One approach to the

problem is to estimate the "distance" between a system and a given

linearization, and choose S' using a variation of the nearest-neighbor

algorithm. When distance is defined as the norm of the vector difference

between the system state and a point of linearization, this approach may

yield a locally optimal method for choosing S'; however, there may be a

linearization which is a large distance from the state estimate, but which

is quite close to the characteristics of the actual system. It would be

nice if there existed a measure of distance which yielded a globally

optimal choice of S'. Whether or not this measure exists is a question

still to be answered.

This research is a specific case of a more general area of problems.

Suppose the researcher has a system with a continuous state space represen-

tation and is interested in some compact subset of the state space. If he

considers a covering for this subset as a finite-state space, with links

between states representing the existence of a path from one element in the

covering to a point within another neighborhood, in what way does optimiza-

tion over this finite-state space correspond to optimization over the

continuous space? Some connections with previous work appear in the

discretization of a state space for digital purposes. Future applications

may include problems such as using this viewpoint to generate good, fast

approximations to the solution of optimal control problems and reliability optimization problems concerned with continuous failure modes in systems.

2. Failure Detection Algorithms

Prof. A.S. Willsky, Dr. S.B. Gershwin, Dr. K.-P. Dunn, Mr. E. Chow, and Mr. R. Bueno have continued the study of failure detection system design methods, as outlined in the preceding progress report. In this time period, Prof. Willsky completed a survey[1] of a number of failure detection methods, outlining their strengths and weaknesses and the various key tradeoffs in the designs. In addition, the GLR method (see the survey paper[1] or the previous progress report) has been studied in great detail. Specifically, an extensive simulation study, using the simplified two-dimensional longitudinal dynamics of the F-8C aircraft, has been completed, and a report describing the qualitative behavior of the detector, false alarm-correct detection tradeoffs, etc., is forthcoming. In this report we also describe a detailed analytical package that has been and is continuing to be developed. This package allows one to calculate probabilities of correct detection, false alarm, mis-classification of failures, and the expected delay time in detection. We have applied these techniques to the F-8C aircraft problem, and have determined several of the relevant tradeoffs between these performance parameters. Further development of these methods will be done in the coming months. Specifically, the following issues are to be considered in the immediate future:

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[1] A.S. Willsky, "A Survey of Design Methods for the Detection of Failures in Dynamic Systems," presented at the M.I.T.- NASA Ames Workshop on System Reliability Issues for Future Aircraft," August 1975; to be submitted for publication.

(1) The determination of the sensitivity of the detector via simulations of the F-8C aircraft with errors in the relevant aero coefficients.

(2) Development of a simplified detection system and a study of its performance via analysis and simulations.

(3) Development of detection methods for other types of failure modes than those presently being explored.

3. Analysis of Nonlinear Filters

The work of Prof. A.S. Willsky and Mr. S.I. Marcus reported in the previous research report has been written up in several papers (see the references at the end of this report). Work in the area of nonlinear stochastic filters has continued along the following lines:

1. We are presently investigating the extension of the optimal nonlinear estimation results of Marcus and Willsky (see the references) to obtain accurate suboptimal estimators for other classes of nonlinear systems.

2. In his thesis[2], Marcus developed a suboptimal estimation technique based on harmonic analysis. This technique is being applied to design estimators for attitude estimation systems.

3. Prof. Willsky and Mr. V. Klebanoff are studying the "harmonic analysis filters" developed by Willsky and Marcus in order to determine analytical measures of their performances.

4. Prof. Willsky and Mr. J. Eterno are considering the problem of the optimal demodulation of a sinusoidal signal. The method here is quite different from the Fourier series methods of Marcus and Willsky.

This new approach involves a direct consideration of the so-called "representation theorem" expression for the conditional probability density. We are presently deriving approximate expressions for the expectations needed to evaluate the density. This approach may prove to be extremely useful in a wide variety of nonlinear estimation applications.

5. Using the same Volterra series methods as those explored by Marcus[2] Prof. Willsky is studying the stability of linear feedback systems containing randomly varying gains.

4. Design of Compensator Incorporating Time Delays

Prof. Willsky and Mr. R. Kwong have undertaken a study of the design of feedback compensators that incorporate time delays. Consider the linear system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) \tag{2}$$

where $x(0)$ is arbitrary. If we restrict ourselves to linear feedback laws

$$u(t) = Kx(t) \tag{3}$$

the best we can hope for is an exponential decay of the output. However, if we allow time delays in the control

$$u(t) = K_1 x(t) + K_2 x(t-h) \tag{4}$$

one can often drive the output to zero (and keep it there) in finite time. In his thesis[3], Mr. Kwong has considered a development of such a design in

2. S.I. Marcus, "Estimation and Analysis of Nonlinear Stochastic Systems," Ph.D. Thesis, Dept. of Electrical Engineering and Computer Science, M.I.T., May 1975.

the single-output case and has devised a procedure for determining K_1 and K_2 when (A,B) is a controllable pair of dimension at least 3. One of the basic results of this study is reported in a brief summary paper.[4]

The next steps in this study are:

(1) The consideration of the dual problem of "deadbeat observers" -- i.e. assuming (A,C) is observable, design a system of the form

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K_1(y(t)-C\hat{x}(t)) + K_2(y(t-h) - C\hat{x}(t-h)) \quad (5)$$

so that the estimation error

$$e(t) = x(t) - \hat{x}(t)$$

is driven to zero in finite time.

(2) Extend Kwong's results to the multiple output case.

(3) Included delays in the system dynamics and output.

(4) Consider the design of output compensators of the form (5) plues

$$u(t) = K_1\hat{x}(t) + K_2\hat{x}(t-h)$$

so that system output is nulled in finite time.

Since many physical systems have inherent time delays, the above study should aid in our overall understanding of these systems and in our ability to devise effective control systems for them.

3. R.H.-S. Kwong, "Algebraic Structure and Estimation of Time Delay Systems," Ph.D. Thesis, Dept. of Electrical Engineering and Computer Science, M.I.T., Cambridge, Mass., Sept. 1975.

4. R.H.-S. Kwong and A.S. Willsky, "Finite-Time Zero Error Control Via Delay Feedback," 1975 IEEE Conf. on Decision and Control, Houston, Dec. 1975.

5. Digital Compensator Design

Prof. Willsky and Mr. G.K. Roberts have continued their efforts in attempting to understand the fundamental limitations of digital implementation of control systems (see the preceding progress report). The basic motivation for this study is to take a first step in developing a control design methodology that directly incorporates computer tradeoffs. Specifically, we have been considering the design of digital controllers for linear systems based on quadratic performance measures. Our study has included explicitly the constraint of a finite time to perform a real multiplication, and we have concentrated on finding low-order examples for which simpler, faster, "suboptimal" control laws outperform the slower "optimal" control, which requires a much slower sampling rate. The eventual aim of this study is to develop concepts for the control of large-scale systems, and our initial ideas will be described in detail in Mr. Roberts' forthcoming thesis.

6. Estimation for Systems with Echoes

A problem of great practical importance is the estimation of signals which may be delayed in time or which may be received together with delayed versions. The simplest such model is

$$y(t) = h_1(x(t)) + h_2(x(t-\tau)) + N(t) \tag{1}$$

where x is the signal to be recovered, N is receiver noise, and τ is a time delay -- either known or unknown. Prof. Willsky and Mr. R. Kwong have studied the estimation of x given such a received signal (actually a more general one involving the possibility of distributed delays) when the delay is known, and

a general representation theorem and the required nonlinear filter equations
are presented in Mr. Kwong's thesis[5].

The problem in the linear case

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} A_i x(t-\tau_i) + w(t) \qquad (2)$$

$$y(t) = Cx(t) + \sum_{i=1}^{N} C_i x(t-\tau_i) + v(t) \qquad (3)$$

is studied in great detail in Kwong's thesis, and the stability of the
optimal steady-state filter is established when there is a single delay in
(3). This result has an important dual, which establishes the stability of
optimal regulators for linear delay systems with quadratic criteria.[5]

Issues to be considered in the near future include:

(1) Design of suboptimal filters based on the optimal, infinite-
dimensional equations derived by Kwong. This is for both the
linear and nonlinear case.

(2) Study of filter stability in the linear case when one allows
multiple delays in (3),(4).

(3) Study of the problem when the time delay in (1) is unknown.

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5. R.H.-S. Kwong, "Algebraic Structure and Estimation of Time Delay Systems,"
Ph.D. Thesis, Dept. of Electrical Engineering and Comp. Sci., M.I.T.,
Cambridge, Mass., Sept. 1975.

7. <u>Low-order Compensator Design</u>

The efforts of Prof. T.L. Johnson, Mr. K. Tong, Mr. B. Llorens, and Mr. Motazedi continue to focus on the design of low-order dynamic compensators for multivariable linear systems. The interplay between formulation of the optimal control problem, existence of solutions, and structure of the optimal control law is the focus of this research.

In a forthcoming ESL Technical Memorandum we will present the solution of the reduced-order observer-based optimal compensator design problem for the standard quadratic performance index. Reduced-order observer-based compensators (i.e., those based on a Luenberger observer which estimates those linear combinations of states required for the optimal input signals) have been studied by Brasch and Pearson, Fortmann and Williamson, Rothschild, and Sarachik, among others. The minimal dimension of such compensators is known to be the maximum of (a) the observability index of (A,C) minus one, and (b) $(n-m)r/m$ where $r=$ number of inputs and $n=$ order of plant, and $m=$ number of outputs. Lower order linear compensators which stabilize the plant may still exist, but they cannot be based on observer theory. The above authors generally use a pole-placement approach as opposed to minimizing the standard quadratic performance index. We show that the optimal structure is indeed given by an observer which estimates (Fx) where F are the Kalman gains for the linear regulator problem, and give an explicit procedure for construction of the optimal compensator. Analogous results were proved by Miller (1972)[6] and Blanvillain (1973)[7] for the $(n-m)$th order observer. We review the literature and provide design methods for any order down to the minimum cited above.

We have used initial-state averaging over a random initial plant state to determine the optimum initial state of such compensators. This method is not entirely satisfactory, and the optimal compensator dynamics in fact depend on the initial state covariance in this case. A large covariance implies faster optimal dynamics. There appears to be an alternative, and somewhat more appealing interpretation of our necessary conditions; an equivalence relation between the random initial state and the stochastic steady state problems can be established for the (n-m) order compensator. In the stochastic steady state problem we assume the same plant and compensator structure, except that the plant is driven by a random (white noise) input and is in stochastic steady state: we seek to minimize the expectation of the integrand of the usual quadratic performance index, which is time-invariant. Perfect observations are assumed. Essentially, the role of the initial state covariance is played by the input noise intensity of this new problem.

The problem of sensitivity analysis and implementation of such designs are still under investigation. We intend to develop a reduced-order design for a realistic aircraft model (see below).

Some authors have claimed that observer-based designs are sensitive to modelling error and also do poorly in regulating steady-state errors in the face of disturbances. They proposed including forward-loop integration in a rather ad-hoc manner which mimics conventional frequency-domain methods. We grant that model sensitivity may be a problem, but expect that (in the absence

6. Miller, R.A., "Specific Optimal Control of the Linear Regulator using a Minimal-Order Observer," Int. J. Control, 1973, Vol. 18, No.1, pp. 139-159.
7. Blanvillain, P., "Optimal Compensator Structure for Linear Time-Invariant Plant with Inaccessible States", ESL Report R-546, May 1974.

of model mismatch) the structure of the limited-dimension compensator should be sufficiently general to allow for forward loop integration to achieve zero steady state error, when required by the quadratic performance index. This demonstration requires a slight generalization of the problem formulation above to allow exogenous inputs to the compensator. We are considering such problems.

We also continue to investigate the question of uniqueness, or the free design parameters of the optimal compensator.

8. Descent-Phase Controller for 4-D Navigation

In a recent thesis by F. Lax we have investigated the feasibility of designing a time-varying control law to achieve simultaneous trajectory following and accurate control of runway-marker arrival time for the descent phase of flight. The procedure for linearizing the nonlinear aircraft equations of motion is well-known, but is seldom carried out in a way consistent with the "LQG design philosophy". In addition, the descent phase of flight is usually handled by gain scheduling models, rather than designing true time-varying optimal gains as espoused by "the" theory. We have carried out all these things in a consistent manner for the longitudinal dynamics of the Boeing 727 on an approach to Logan Airport. The time-varying linearized controller was furthermore implemented on the nonlinear digital simulation and pilot-tested under various mean-wind conditions on the ESL cockpit simulator. The results were quite encouraging and will be reported in a forthcoming ESL Report.

9. Infinite Dimensional Mathematical Programming Problems and Optimal Control Problem with Constraints

There recently has been considerable progress in non-linear programming, both theoretical and computational. It is recognized that duality plays a

central role and dual methods may turn out to be the most important computationally. The use of augmented Lagrangians [1] is expected to play a key role.

Open loop optimal control problems are infinite-dimensional mathematical programming problems. However, they have considerable structure (namely, the time structure) which must be exploited. Viewing them as infinite dimensional mathematical programming problems has the advantage one can focus on issues which might not be evident otherwise. It has long been felt that a synthesis of mathematical programming and optimal control would be an important development. Furthermore, it has also been felt that the so-called direct methods (Rayleigh-Ritz), which have been largely neglected in computational work for optimal control problems, should allow us to use mathematical programming in an efficient way (that is, exploiting the time structure).

In recent work ([2]-[5]), we feel we have taken important steps in this direction. More precisely, we have given a reasonably complete duality theory for convex optimal control problems and also shown how the dual problem can be efficiently solved using the Ritz method.

Work is now progressing in several directions:

a) Using similar ideas we are investigating the following linear problem:

Minimize

$$c(x,u) = \int_0^1 (\alpha'x(t) + \beta'u(t))dt$$

subject to

$$\dot{x} = A(t)x(t) + B(t)u(t)$$

$$K_c u(t) \leq 0 , \quad K_s x(t) \leq 0.$$

We conjecture that without a Slater type condition and with feasibility of the primal and dual problems that there is no duality gap. This result will generalize an earlier result of Levinson [7] and will have great computational significance. Problems of this type arise, for example, in Dynamic Leontief models and in certain problems of communication networks.

b) After discretization of the dual problem using finite element methods we are left with a mathematical programming problem. In the quadratic case we are left with a quadratic programming problem. It appears however that these quadratic programming problems should not be solved using complementarity theory. We are investigating the structure of the mathematical programming problems with a view to obtaining efficient techniques (perhaps decomposition methods) for their solution.

References

1. R.T. Rockafellar, "Augmented Lagrange Multiplier Functions and Duality in Convex Programming," SIAM J. on Control, 12 (1974) pp.268-285.

2. W.W. Hager, "Rates of Convergence for Discrete Approximations to Problems in Control Theory," Ph.D. Thesis, Mathematics Department, M.I.T., June 1974.

3. W.W. Hager and S.K. Mitter, "Lagrange Duality Theory for Convex Control Problems," to appear in SIAM J. on Control.

4. W.W. Hager, "The Ritz-Trefftz Method for State and Control Constrained Optimal Control Problems," to appear in SIAM J. on Numerical Analysis.

5. W.W. Hager, "Rates of Convergence for Discrete Approximations to Unconstrained Control Problems", to appear in SIAM J. on Numerical Analysis.

6. R.T. Rockafellar, "State Constraints in Convex Control Problems of Bolza," SIAM J. on Control, 10 (1972), pp. 691-715.

7. N. Levinson, "A Class of Continuous Linear Programming Problems". J. of Math. Analysis and Applications, 16, (1966), pp. 73-83.

10. Robust Compensator Design

Mr. P.K. Wong, Mr. M. Safanov, and Prof. M. Athans have been investigating several aspects for design of [   ]-Quadratic Control System design and Kalman filter designs that are [   ]st under large parameter variations.

In the design of compensators for the control of dynamical systems, it is common engineering practice to introduce the assumption that the dynamical system to be controlled can be adequately modeled by a finite set of ordinary linear differential equations. Under this assumption, the engineer can, with relative ease, synthesize a linear state-feedback compensator for the model which will minimize a quadratic performance index. The class of compensators produced by this design procedure have come to be generally known as linear-quadratic- (LQ) controllers. The research carried out is aimed at determining criteria by which it will be possible to judge _a priori_ whether this design prodecure can be expected to yield good results for any particular dynamical system.

Results reported in the literature to date on the validity of this design procedure have been largely limited to a posteriori tests of the sensitivity of the controlled system to infinitesimal perturbations in the various parameters describing the system. However, very little is known quantitatively about the validity of the use of LQ controllers. One notable exception is that _a priori_ quantitative bounds have been found for the minimum gain and phase margin of single-input/single-output linear systems with LQ controllers. Recently, the gain margin (but not the phase margin) bound for LQ controllers has been generalized by P.K. Wong to include

multi-loop linear feedback systems.

We are presently examining analytical techniques which are expected to make possible more widely applicable quantitate statements about the validity of the use of LQ controllers. Currently, efforts are being directed to the generalization of the aforementione notations of *a priori* gain and phase margin to LQ controllers applied to nonlinear systems with multiple feedback loops. The immediate objectives are to

(1) provide a theoretical foundation for the empirical fact that LQ controllers tend to be insensitive to modeling error,

(2) provide engineers with means for determining *quantitatively* the magnitude of modeling error which can be tolerated before it is necessary to question the reliability of a system with an LQ controller, and

(3) permit better understanding of how such effects as phase shift, time-delay, and saturation can be expected to affect the performance of multi-loop LQ control systems.

Preliminary results indicate that a quantitate bound on the amount of nonlinearity and phase shift which can be tolerated in the feedback loops of systems with LQ controllers can be determined *a priori* for entire class of systems with LQ controllers.

The results for robust LQ controllers can also provide valuable insight, namely the design of Kalman filters for which the covariance of the fake white noise can play the role of a design variable. In the long run we hope to obtain a unified quantitative design procedure for the overall

LQG robust design concept.

Faculty M. Athans

Research Assistants

P.K. Wong
M.G. Safonov

Reference: P.K. Wong "On the Interaction Structure of Feedback Control Systems", S.M. Thesis, M.I.T., August 1975; also reput ESL-R-625, September 1975.

11. Numerical Methods for the Lyapunov Equation

The importance of the algebraic Lyapunov equation in problems of steady state estimation and control of time-invariant linear systems is well-known. Mr. T. Athay and Prof. N.R. Sandell Jr. have initiated a research effort aimed at development and careful numerical analysis of algorithms for Lyapunov equation solution. The research will be documented in the forthcoming Engineer's Thesis of T. Athay [1]. The following preliminary results have been obtained.

The notion of conditioning is basic to numerical analysis. We have obtained a precise notion of conditioning for the Lyapunov equation. We have been able to demonstrate that the often heard statement that systems with widely spread eigenvalues have ill-conditioned Lyapunov equations is correct in a precise sense, and moreover is a property of the equation and not the solution technique.

Although different algorithms for Lyapunov equation solution cannot affect the problem's inherent conditioning, they certainly will have different

error properties and operation counts. We have analyzed a number of the more popular algorithms with respect to these properties.

Finally, motivated by the block coupled structures commonly found in large scale engineering systems, we have developed certain algorithms that explicitly exploit these structure. Analysis and numerical test of these algorithm is presently underway.

References:

1. T. Athay, Numerical Analysis of the Lyapunov Equation with Application to Interconnected Power Systems.

12. Perturbation Methods in Linear Filtering and Control

Large scale aerospace and other engineering systems are characterized by interactions of smaller subsystems [1]. These interactions may be weak so that an approximate analysis of the decoupled subsystems may be employed, or they may be so strong that certain subsystems can be aggregated into an equivalent subsystem with fewer state variables. The mathematical tools of perturbation theory are an appropriate means of analysis for such situations.

Mr. D. Teneketzis and Prof. N.R. Sandell Jr. have initiated research aimed at extending the existing theory of perturbation methods for deterministic optimal control problems to stochastic filtering and control problems. The research will be documented in the forthcoming Master's Thesis of D. Teneketzis [2]. The following results have been obtained:

For the weak coupling problem, the analysis proceeds via regular (non-singular) perturbations. Conditions have been obtained for the stability of filters designed for the perturbed (decoupled) subsystems but applied to

the actual, coupled system and computable bounds have been obtained for the resulting degradation in performance. Analogous results have been obtained for the stochastic control problem.

For the strong coupling problem, the analysis proceeds via singular perturbation theory. An interesting and nonintuitive result characterizing the behavior of the optimal filter as the coupling becomes infinitely strong has been obtained, and the analysis has been extended to systems with three levels of coupling.

[1] N.R. Sandell Jr., "Information Flow in Decentralized Systems," Conference on Directions in Decentralized Control, Many-Person Optimization and Large-Scale Systems, Sept. 1975, Cambridge, Mass.

[2] D. Teneketzis, Perturbation Methods in Stochastic Control, M.S. Thesis, forthcoming.

E. FINANCIAL STATUS

As of December 1, 1975, approximately 30% of the grant funds ($70,000)
were expanded corresponding to 25% of the grant period.

No consideration for cost sharing is included in the above figures.

F. <u>PUBLICATIONS</u>

The publications below describe research reported in this or previous

progress reports and supported all or in part by NASA grant NGL-22-009-124.

The references cited represent new material generated or published since the

last progress report.

Copies have been transmitted to the NASA Technical Information Center

and to Mr. Brian F. Doolin at NASA/AMES Research Center.

<u>Papers Under the NASA Grant</u>

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   of Linear Systems Containing Colored Multiplicative Noise", <u>IEEE Trans.
   on Automatic Control</u>, Vol. AC-20, Oct. 1975, pp. 711-713.

2. S.I. Marcus and A.S. Willsky, "Algebraic Structure and Finite Dimensional
   Nonlinear Estimation," <u>Proc. of the Symposium Algebraic System Theory</u>,
   Udine, Italy, June 1975.

3. A.S. Willsky, "A Survey of Deisgn Methods for the Detection of Failures
   in Dynamic Systems," presented at the M.I.T.- NASA Ames Workshop on System
   Reliability Issues for Future Aircraft, August 1975; to be submitted for
   publication.

4. S.I. Marcus and A.S. Willsky, "Estimation for Certain Nonlinear Systems,"
   <u>Seminaires IRIA</u>, Rocquencourt, France.

5. R.H.-S. Kwong, "Structural Properties and Estimation of Delay Systems,"
   Ph.D. Thesis, Dept. of Elec. Eng. and Comp. Sci., M.I.T., Sept. 1975.

6. R.H.-S. Kwong and A.S. Willsky, "Finite-Time Zero Error Control for Linear
   Continuous Time Systems Via Delay Feedback," 1975 IEEE Conf. on Decision and
   Control, Houston, Texas, Dec. 10-12, 1975.

7. A.S. Willsky, J.J. Deyst, and B.S. Crawford, "Two Self-Test Methods Applied
   to an Inertial System Problem," <u>J. Spacecraft and Rockets</u>, Vol. 12, No.7,
   July 1975, pp. 434-437.

8. L.K. Platzman and T.L. Johnson, "A Linear-Quadratic Gaussian Control
   Problem with Innovations-Feedthrough Solution", submitted to IEEE Trans.
   Auto. Contr., Sept., 1975.

9. F.M. Lax, "Design and Simulation of a Descent Controller for Strategic 4-Dimensional Aircraft Navigation", S.M. Thesis, Sept., 1975.

10. K. Tong and T.L. Johnson, "Reduced-order Observer Based Compensators", ESL Technical Memorandum, to appear.

11. P.K. Wong, "On the Interaction Structure of Feedback Control Systems," S.M. Thesis, M.I.T., August 1975; also Report ESL-R-625, Sept. 1975.

12. T. Athay, Numerical Analysis of the Lyapunov Equation with Application to Interconnected Power Systems.

13. N.R. Sandell, Jr., "Information Flow in Decentralized Systems," Conf. on Directions in Decentralized Control, Many-Person Optimization, and Large-Scale Systems, Sept. 1975, Camb., Mass.

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