



Determining Aircraft Moments of Inertia from Flight Test Data

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Outline



- **Introduction**
- **Modeling Approach**
- **Flight Test Maneuver**
- **F-16 Nonlinear Simulation Results**
- **E1 Flight Test Results**
- **Summary**

Introduction



Uses for Aircraft Moments of Inertia:

- Nondimensional aerodynamic modeling
- Simulation and dynamic analysis

Conventional Methods Used:

- Ground testing
- Computer-Aided Design (CAD)
- ***Costly and time-consuming***

Can aircraft moments of inertia be determined accurately from flight test data?





Modeling Approach

Nonlinear Rotational Equations of Motion

Pitch:

$$\dot{q} = \textcircled{M} + \frac{(I_z - I_x)}{I_y} pr + \frac{I_{xz}}{I_y} (r^2 - p^2) + \cancel{\frac{I_p}{I_y} \Omega_p r} \approx 0 \text{ for idle power}$$

Yaw:

$$\dot{r} = \textcircled{N} + \frac{(I_x - I_y)}{I_z} pq + \frac{I_{xz}}{I_z} (\dot{p} - qr) - \cancel{\frac{I_p}{I_z} \Omega_p q} \approx 0 \text{ for idle power}$$



Equation-Error Modeling

$$\dot{q} = \left[M_o \bar{q} + M_\alpha \bar{q} \alpha + M_q \bar{q} \frac{q}{V} + M_{\delta_e} \bar{q} \delta_e \right] + c_3 pr + c_4 (r^2 - p^2)$$

$$\dot{r} = \left[N_o \bar{q} + N_\beta \bar{q} \beta + N_p \bar{q} \frac{p}{V} + N_r \bar{q} \frac{r}{V} + N_{\delta_a} \bar{q} \delta_a + N_{\delta_r} \bar{q} \delta_r \right] + c_5 pq + c_6 (\dot{p} - qr)$$

$$c_3 = (I_z - I_x) / I_y \quad c_4 = I_{xz} / I_y$$

$$c_5 = (I_x - I_y) / I_z \quad c_6 = I_{xz} / I_z$$



Flight Test Maneuver

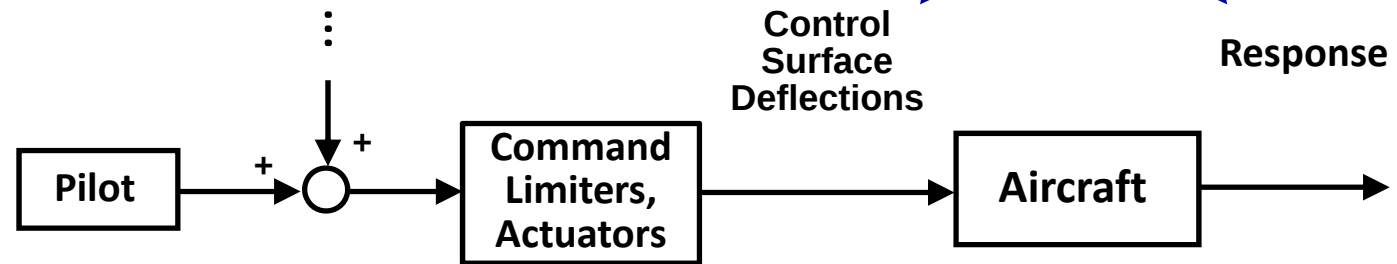
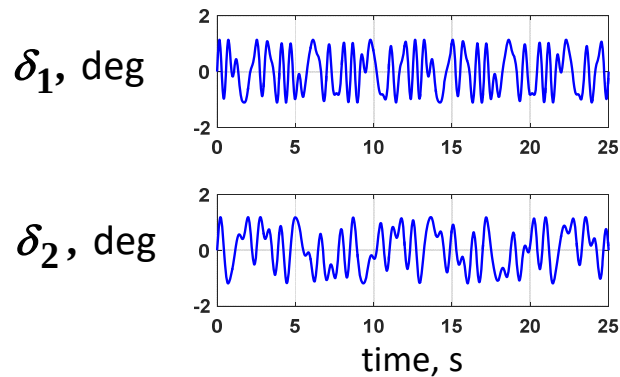
- High angular rates to make the nonlinear inertia terms significant
- Idle power to make propulsion gyroscopic terms negligible
- Low nominal angle of attack for linear aerodynamics
- Control surface excitations for decorrelated explanatory variables

Rolls into descending turns at idle power and low nominal angle of attack with automated multi-axis orthogonal perturbation inputs on the control surfaces



Flight Test Maneuver

Programmed
Test Inputs
(PTI)



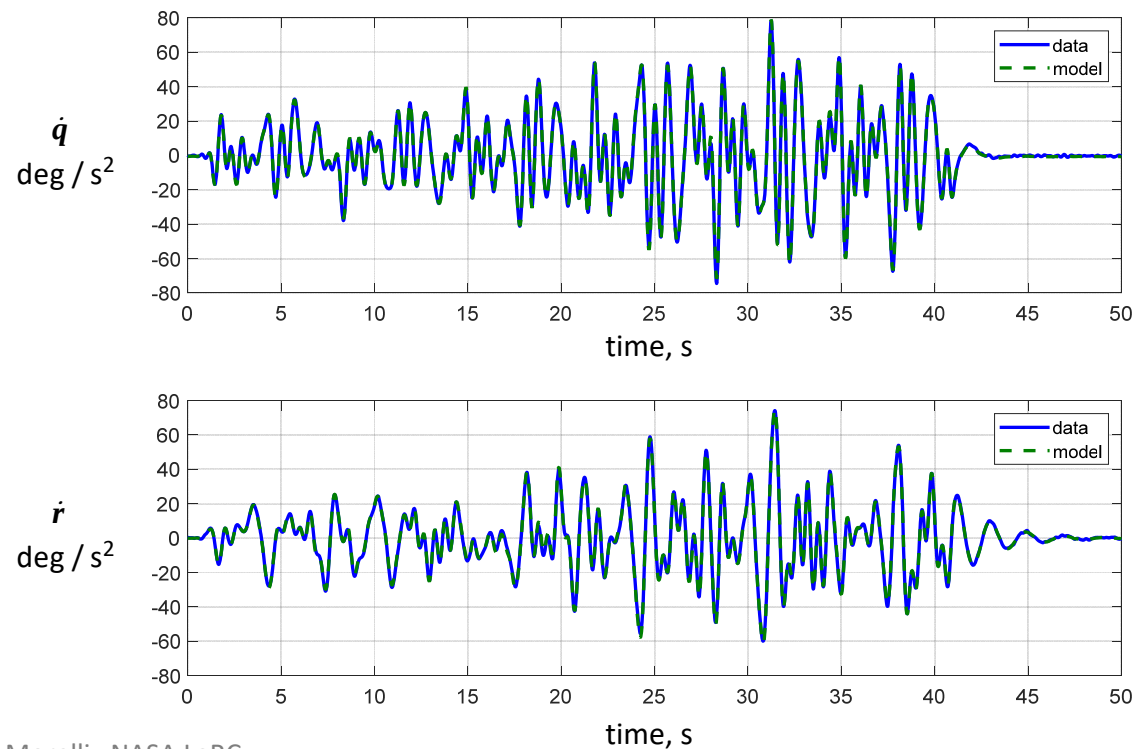
F-16 Nonlinear Simulation: Desktop joystick

E1 Flight Test: Radio Control from the Ground

F-16 Nonlinear Simulation Results



Single maneuver, alternating rolls into descending turns with PTI active



F-16 Nonlinear Simulation Results



Linear Estimation

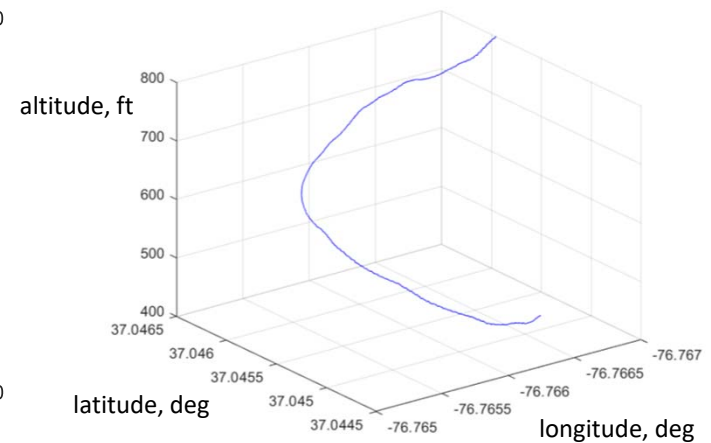
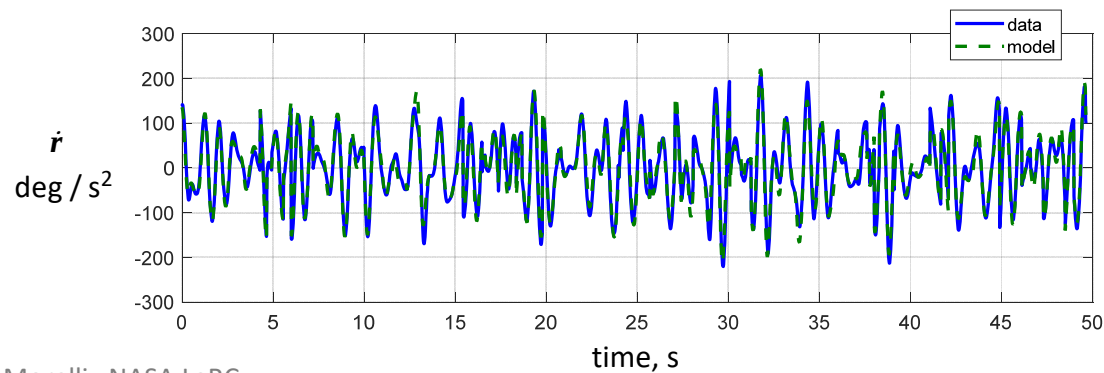
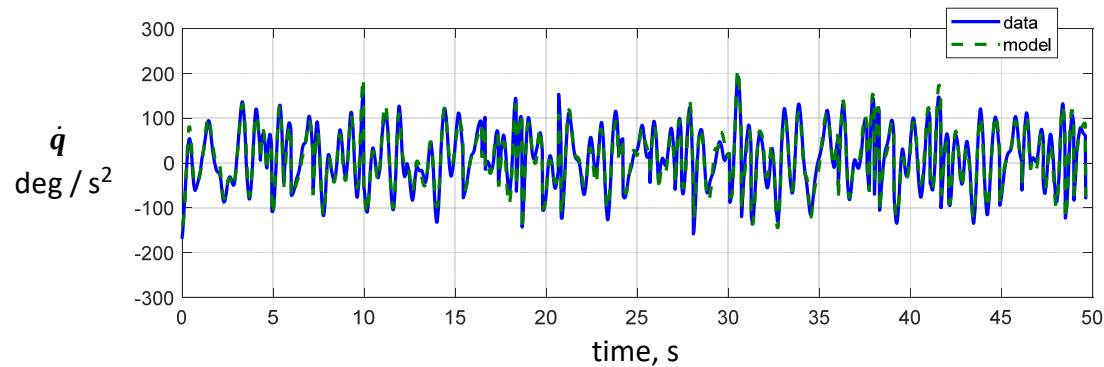
Parameter	True	Estimate $\pm$ Standard Error	Percent Error
c_3	0.9604	0.9643 ± 0.0234	0.40
c_4	0.0176	0.0166 ± 0.0019	-5.68
c_5	-0.7340	-0.7517 ± 0.0189	-2.41
c_6	0.0156	0.0147 ± 0.0011	-5.25



E1 Flight Test Results



Five maneuvers, descending turns with PTI active, $\alpha \leq 7$ deg



E1 Flight Test Results



Nonlinear Estimation

Parameter	Ground Test Estimate*	Flight Test Estimate $\pm$ Standard Error	Percent Error
I_x	2.964	3.053 ± 1.002	3.01
I_y	8.776	8.711 ± 0.732	-0.74
I_z	11.716	11.036 ± 0.495	-5.80
I_{xz}	0.750	0.779 ± 0.107	3.89



* Ground test uncertainty $\leq 3 \%$



Summary:

A novel flight test method for accurately determining moments of inertia was developed and demonstrated

Key Findings:

- Large-amplitude piloted flight test maneuvers at idle power with multi-axis orthogonal excitations produced informative flight data for accurately determining aircraft moments of inertia
- Simple and versatile equation-error model formulation produced accurate results for both F-16 nonlinear simulation and E1 flight test
- Costly and time-consuming ground testing or CAD modeling can be avoided



Questions?

Backup Slides



Equation-Error Solution



- Data can be selected from multiple maneuvers and need not be contiguous in time
- Linear estimation for inertia constants c_3, c_4, c_5, c_6
- Nonlinear parameter estimation for inertia tensor elements I_x, I_y, I_z, I_{xz}
- Modeling function data are smoothed for modeling accuracy
- Straightforward solutions using SIDPAC

Orthogonal Optimized Multisine Inputs

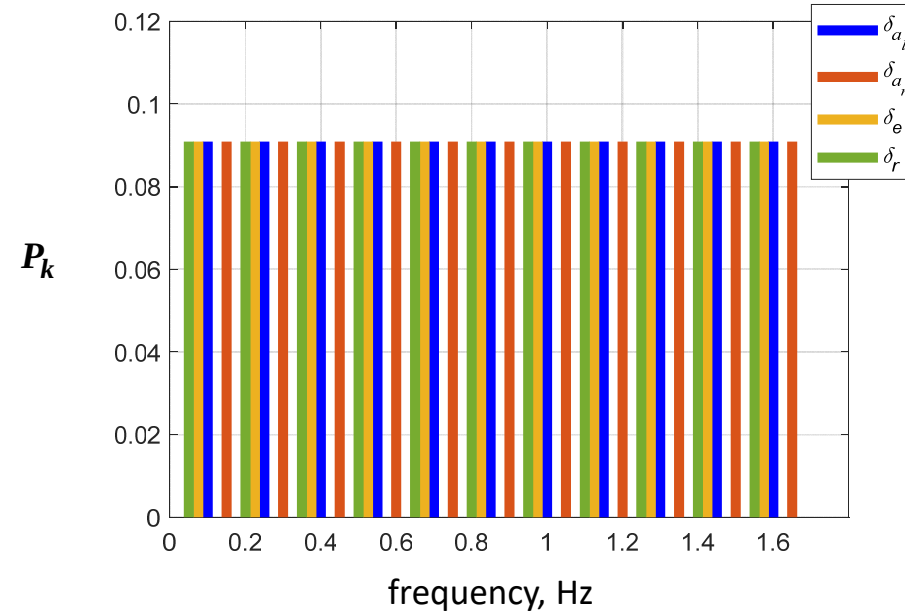


**Programmed Test Input
(PTI) Design**

$$u = \sum_{k \in \{1, 2, \dots, M\}} A_k \sin\left(\frac{2\pi k t}{T} + \phi_k\right)$$

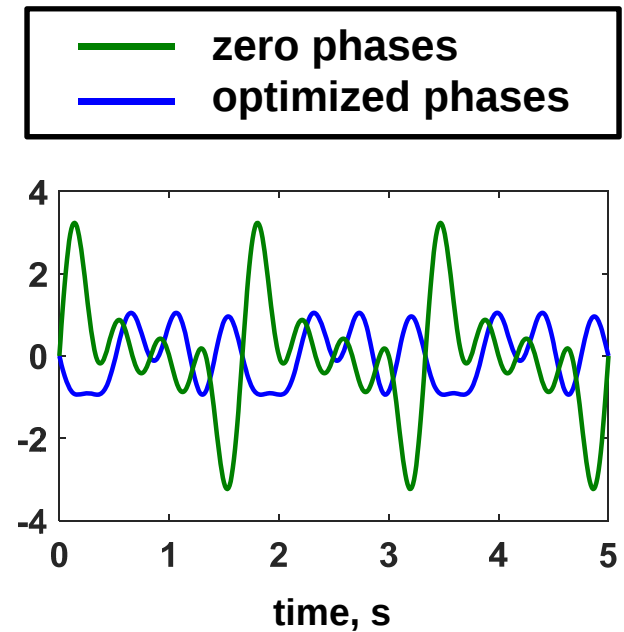
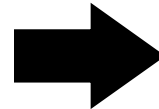
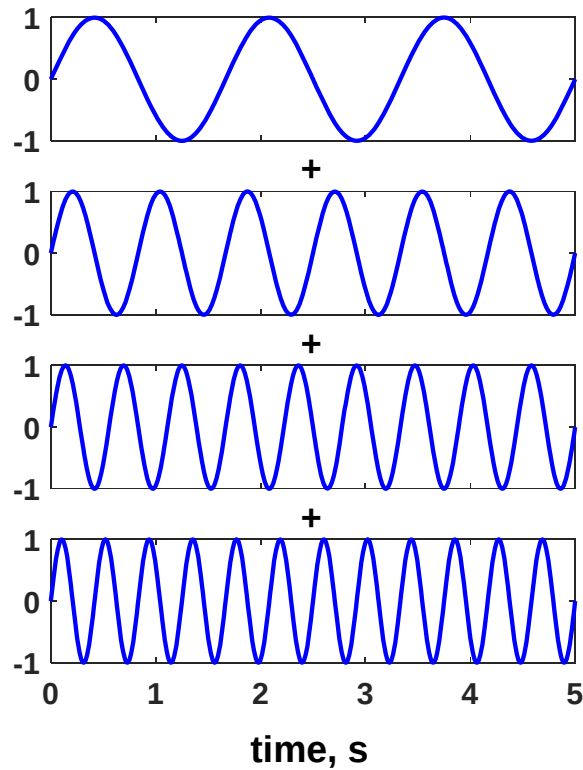
T = maneuver length, s

- Time/Frequency orthogonal
- Simultaneous multi-input
- Robust to various dynamics
- Balanced, minimal excursions
- Easy to design
- Successful in past flight tests



Phases ϕ_k optimized for minimum peak-to-peak amplitude of the sum

Multisine Phase Optimization



Orthogonal Optimized Multisine Inputs



- Inputs are mutually orthogonal in the time and frequency domains
 - Flexibility to model in time or frequency domain
 - Simultaneous input excitation for high efficiency
 - Decorrelates explanatory variables and provides good coverage
- Phases optimized for minimum perturbation amplitudes and the inputs are balanced – maneuver does not move aircraft off nominal trajectory
- Wide-band frequency content for robustness and varied flight conditions
- Easy to design, successful on many other flight tests