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Optimal Input Design for Aircraft Stability and Control Flight Testing

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Abstract Flight testing to characterize aircraft stability and control is a complex and expensive task that involves restrictive practical constraints. Methods are examined for optimizing the inputs applied to aircraft controls during flight test maneuvers to produce informative flight data. Theory underpinning optimal input design for stability and control flight testing is explained. The merits and limitations of optimal inputs are discussed, along with optimization approaches and practical considerations involved in applying optimal inputs in a real flight test environment. Simulated and flight test case studies of optimal multiple-input design for aircraft stability and control flight testing are presented and discussed.

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1 Introduction

An aircraft in flight is a complex nonlinear dynamic system with multiple inputs and multiple outputs. A common simplification applied in aircraft stability and control flight testing is to design the flight test maneuver for small input and output amplitude excursions relative to a reference flight condition, so that a linear dynamic model with constant parameters can be used to characterize the aircraft dynamics. A collection of these locally valid linear models at various flight conditions can be combined to produce a

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global model valid over a large portion of the flight envelope. Global models are useful for applications such as flight simulation, feedback control system design, pilot training, mission rehearsal, and dynamic analysis. However, locally valid linear models are also useful individually for feedback control design, flight envelope expansion, and assessments of aircraft stability, control, and flying qualities at specific flight conditions.

Although successful work has been done recently on global flight test maneuvers and global modeling [1, 27–29, 33], the majority of practical stability and control flight testing involves identifying local linear models from flight test data collected at selected reference flight conditions. Naturally, the flight test maneuver design is crucial for generating informative flight data for accurate modeling. Each flight test maneuver is executed by moving the aircraft controls during the time available for the maneuver. Excitation inputs designed for this purpose are perturbation time series added to the nominal control positions required to achieve the reference flight condition. In this work, the term "input" will be used to refer to the excitation time series, whereas the term "control" will refer to physical control effectors on the aircraft, such as elevator, aileron, or rudder. The topic of this paper is the optimization of input perturbation time series to realize dynamic flight test maneuvers that produce informative flight data for accurate local linear dynamic modeling. Because parameters in local linear dynamic models are used to characterize aircraft stability and control [33], this is often called stability and control flight testing.

Optimizing inputs for aircraft stability and control flight testing is worthwhile because flight testing is expensive, and because many local linear models are required to cover a typical flight envelope. However, the input optimization problem is complicated by aircraft input and output constraints, available maneuver time, the number of controls, outputs, and model parameters, noise and correlations in the data, and the manner in which the aircraft controls can be moved in flight, among other considerations.

Optimal input design for general dynamic systems has a rich literature that dates back to the early 1960s, as described in reviews of the relevant literature [13, 15, 16]. Most early work used a simple linear dynamic model and variational calculus for single-input optimization. The more specialized problem of optimal input design for aircraft stability and control flight testing has been studied extensively, as evidenced by the list of references. The purpose of this paper is to provide theory and background information for optimal inputs applied to aircraft stability and control flight testing, and to demonstrate practical approaches to optimizing multiple inputs for this important problem using simulation and flight test case studies.

Current flight-test practice still commonly employs heuristic single-input designs, such as the impulse, doublet, multistep, and frequency sweep inputs shown in Fig. 1.

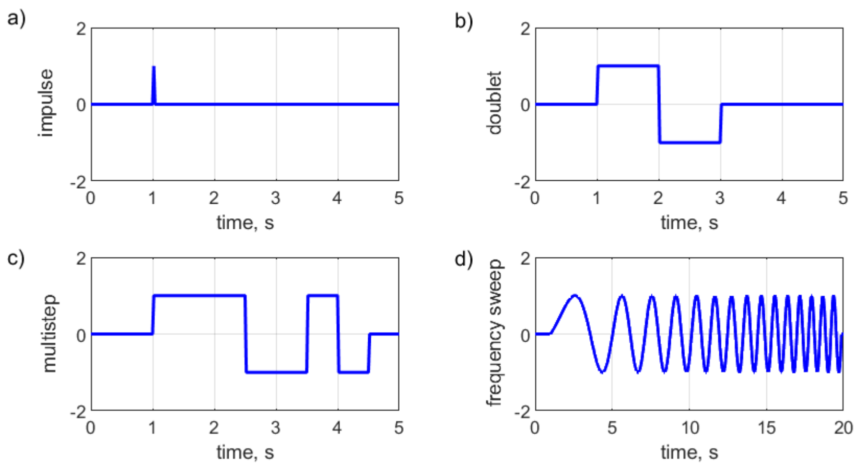

Fig. 1: Heuristic single inputs: a) impulse, b) doublet, c) multistep, d) frequency sweep

Optimal input design specifically for aircraft stability and control flight testing has been done for single-input, multiple-output dynamic systems with multiple model parameters in both the time domain [7,18,41] and the frequency domain [7,8,16–18], using variational calculus and analogies to optimal experiment design for linear regression models. Other work of this kind involved choosing the switching times for a single bang-bang input to optimize the match of the input spectrum to a desired spectrum based on sensitivities computed using a prior model [38]. Because the problem involves multiple outputs and multiple parameters, the cost function for input optimization was defined as a scalar function of the information matrix, as will be explained later.

Notable later developments include suboptimal multiple-input design using Walsh functions [2], optimal multiple-input design using weighted sums of sinusoids [36,42], and applying dynamic programming to design globally optimal multistep inputs for prior models with arbitrary complexity while accounting for practical flight test constraints [20,32]. The latter technique has been successfully demonstrated in flight [3,14,22,24,37]. The same general idea has been used with two-step optimization methods [10,12] and with modification to account for uncertainty in the prior model [9]. Another general approach is optimizing switching times for multiple multistep inputs, which has been done using a gradient method [39] and a genetic algorithm with a Haar wavelet transform [15].

All of these approaches require a prior model to design the optimal inputs, as will be discussed in detail later. This leads to the circular problem of needing a good prior model to design an optimal input for an experiment that will produce data for identifying an accurate model. A cynical view of this situation is that an effective optimal input can only be designed when it is not needed, i.e., when a good prior model is available. This problem has been recognized for a long time, and has been addressed by examining the effect of errors in the prior model on the effectiveness of the optimal input design [3,20], and by proposing an iterative bootstrap approach [23]. An alternative approach is to develop optimal input design methods that do not require a prior model. Such methods can be very practical and effective.

An important advance in designing multiple optimal inputs without a prior model is the orthogonal optimized multisine input design [25,26,33]. This optimal input design has been successfully flown on many different aircraft in a wide variety of flight conditions [33]. The optimization is based on characteristics of the inputs, not on data information computed using a prior model, which is an important practical advantage. This input design technique produces multiple optimal inputs that are mutually orthogonal in both the time domain and the frequency domain at the same time, so that multiple inputs can be applied simultaneously in a wide variety of flight test situations, such as aircraft with many control surfaces [25,27,33,35] or aeroelasticity [6], and at nominal or unusual flight conditions such as high angles of airflow incidence [27] and hypersonic speeds [26,34]. The technique is also very well-suited for real-time modeling [5,27–29,31,33], global aerodynamic modeling [1,27–29,33], and modeling using frequency responses [4].

The purpose of this paper is to show how optimization can be applied to input design for aircraft stability and control flight testing. The next section explains optimal input design based on a prior model. Optimal input design without a prior model is discussed in Section 3. Section 4 shows case studies using an F-16 aircraft simulation and the X-43A hypersonic stability and control flight testing. Conclusions are given in Section 5.

Results shown in the paper were generated using a software package called System IDentification Programs for AirCraft (SIDPAC), which was written in MATLAB® and developed at NASA Langley Research Center. SIDPAC is associated with [33] and is available to US citizens, companies, and organizations [11].

2 Optimal Input Design Based on a Prior Model

Optimal input design will be described first for a simple single-input problem. Then the formulation will be elaborated for the practical aircraft problem with constraints.

2.1 Single-Input, Single-Output Linear Dynamic System with a Single Parameter

A single-input, single-output linear dynamic system with a single model parameter θ can be described by

$$\begin{aligned} &\dot{x}(t) = \theta x(t) + u(t) \qquad x(0) = 0 \\ &y(t) = x(t) \\ &z_i = y_i + v_i \qquad i = 0, 1, \ldots, N-1 \end{aligned} \tag{1}$$

where the i subscript indicates a discrete sampled value at $i\Delta t$ for a constant sampling interval Δt in seconds. The variable x is the state, u is the input, y is the model output, z is the measured output, and v is the measurement noise.

The objective is to design an input time series u that will produce data with optimal information content for accurately estimating the unknown parameter θ. The data consist of discrete measurements of the input (u_i) and output (z_i) over the maneuver length $T = (N-1)\Delta t$ seconds.

The model output y is influenced by the input u through the dynamic model equations (1). If the sensitivity of the model output to the parameter θ (called the output sensitivity, $\partial y/\partial\theta$) has a large magnitude, then the parameter value must be precisely specified if the model output y is to match the measured output z as well as possible. Stated another way, a large output sensitivity magnitude means that the best value of θ can be specified very precisely, given that the model output must match the measured output. On the other hand, if the output sensitivity has a low magnitude, then changes in θ produce only small changes in the model output, and the best value of θ cannot be determined precisely. In the (impractical) limiting case where the output sensitivity is zero, the value of θ cannot be determined at all.

For accurate model parameter estimation based on measured data, the output sensitivity must also stand out above the noise level. Based on these considerations, the cost function for optimal input design in this simple case is:

$$J(u) = \frac{1}{\sigma^2}\int_0^T \left(\frac{\partial y}{\partial \theta}\right)^2 \mathrm{d}t \approx \frac{1}{r^2}\sum_{i=0}^{N-1}\left(\frac{\partial y_i}{\partial \theta}\right)^2 \tag{2}$$

where r^2 is the discrete measurement noise variance, and is related to the continuous-time measurement noise variance σ^2 by $r^2 = \sigma^2/\Delta t$ [40]. Roughly speaking, the expression on the right side of (2) is the squared signal-to-noise ratio for the parameter estimation problem, integrated over the maneuver time T. This quantity is also the inverse of the parameter estimate variance using an unbiased and efficient parameter estimation technique [33]. For this simple problem, maximizing the data information for parameter estimation is equivalent to minimizing the uncertainty in the model parameter estimate based on the measured data.

The output sensitivity can be computed from the dynamic model equations (1) using finite differences or by solving the sensitivity equations [33]:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial x}{\partial \theta}\right] &= \theta\frac{\partial x}{\partial \theta} + x \qquad \frac{\partial x}{\partial \theta}(0) = 0 \\ \frac{\partial y}{\partial \theta} &= \frac{\partial x}{\partial \theta} \end{aligned} \tag{3}$$

The dynamic equation for the output sensitivity is driven by the state x, which is influenced by the input u via the dynamic equations (1). If the parameter θ had multiplied u in the dynamic equation, then the output sensitivity dynamic equation (3) would be driven by the input u directly. Optimal input design for a simple single-input, single-output dynamic system with a single model parameter can be done using variational calculus to find the input time series that optimizes the cost function in (2):

$$u^* = \max_{u \in \mathrm{U}} J(u) \tag{4}$$

where U is the set of all possible input sequences compliant with maximum amplitude and time constraints, and u^* is the optimal input.

The cost function in (2) depends implicitly on the time length T and the input u, as well as the assumed model (1). If there were no practical constraints, then the cost function in (2) could be made arbitrarily large with a large amplitude for u and/or long time length T. However, both the input amplitude and time length are limited in practice, and this is the genesis of the input optimization problem.

In the literature, the maneuver time length T is usually chosen and fixed, and the input amplitude constraint is implemented indirectly using

$$\int_0^T u^2 \mathrm{d}t \approx \sum_{i=0}^{N-1} u_i^2 \Delta t \leq E \tag{5}$$

where E is a chosen maximum value for the input energy. This input energy constraint is used because it is straightforward to implement in an input optimization based on variational calculus. However, Fig. 2 shows that inputs with very different time lengths and amplitudes can have the same input energy calculated from (5). In fact, the real practical constraint is a time-amplitude box that defines the limits for any candidate input time series. Input amplitudes are limited by mechanical stops, flight control software limiters, and/or model structure validity, and the maneuver time is constrained by economics, airspace, and/or the ability to maintain the desired flight condition. Furthermore, the time series candidates must begin and end at zero, because perturbation inputs are applied additively to the nominal control deflections necessary to achieve the desired reference flight condition for the maneuver.

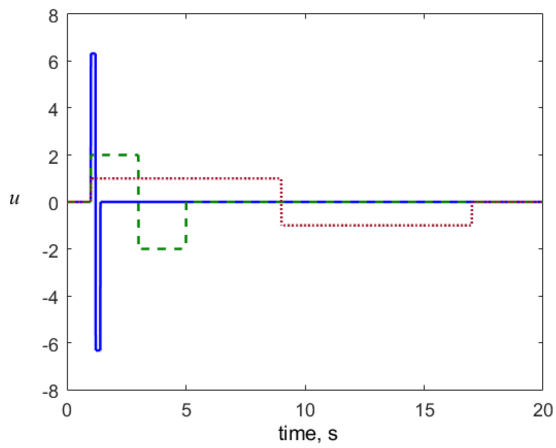

Fig. 2: Doublet input variations with constant energy

From (1) and (3), the input time series is nonlinearly connected to the output and output sensitivity by the convolution integral solutions to the associated linear differential equations. Furthermore, the expression for the cost function in (2) is a time integral that is nonlinear in the output sensitivity. These are further reasons that a fair comparison of input designs means that all input time series being compared must be confined within the same time-amplitude constraints.

2.2 Single-Input, Multiple-Output Linear Dynamic System with Multiple Parameters

The input optimization problem gets much more complex for multiple outputs and more than one model parameter, which is a typical situation for aircraft problems. In that case, the model parameters appear in the linear system matrices for the dynamic model equations, and the state x, output y, and measurement z are vectors:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \qquad x(0) = 0 \\ y(t) &= Cx(t) + Du(t) \\ z_i &= y_i + v_i \qquad i = 0, 1, \ldots, N-1 \end{aligned} \tag{6}$$

The data information content in a maneuver is quantified by the information matrix M:

$$M = \int_0^T \left(\frac{\partial y}{\partial \theta}\right)^T \Sigma^{-1} \left(\frac{\partial y}{\partial \theta}\right) \mathrm{d}t \approx \sum_{i=0}^{N-1} \left(\frac{\partial y_i}{\partial \theta}\right)^T R^{-1} \left(\frac{\partial y_i}{\partial \theta}\right) \tag{7}$$

where $R = \Sigma/\Delta t$ is the discrete measurement noise covariance matrix, quantifying the different noise levels for the multiple outputs. Typically, measurement noise sequences are mutually uncorrelated, so that the R matrix is diagonal. For input design, the information matrix is set equal to the discrete approximation from (7),

$$M = \sum_{i=0}^{N-1} \left(\frac{\partial y_i}{\partial \theta}\right)^T R^{-1} \left(\frac{\partial y_i}{\partial \theta}\right) \tag{8}$$

Input optimization is subject to amplitude constraints on the inputs, but also to approximate amplitude constraints on the outputs, to maintain the flight condition and the validity of the assumed model structure. The input amplitude constraints are relatively easy to implement, but the output amplitude constraints are not, because the outputs depend on the amplitude and frequency content of the input acting through the dynamic system, which is characterized only approximately by the prior model.

As a result of the multiple outputs and multiple model parameters, the data information is characterized by a matrix, so that a choice must be made as to what scalar function of the information matrix should be used as the input optimization criteria. Some of the options and their associated implications are discussed in [16,33,38]. The most common approach is to minimize some scalar function of the inverse information matrix, called the dispersion matrix D:

$$D = M^{-1} \tag{9}$$

The dispersion matrix D is the parameter estimate covariance matrix obtained using an unbiased and efficient parameter estimator [33], analogous to the single-output, single-parameter case. This is called the Cramér-Rao lower bound. The diagonal elements of D are the theoretical minimum values of individual parameter variances. The Cramér-Rao lower bounds for individual parameter standard errors are the square roots of the diagonal elements of the dispersion matrix:

$$s\left(\theta_j\right) = \sqrt{d_{jj}} \qquad j = 1, 2, \ldots, n_p \tag{10}$$

where d_{jj} are the diagonal elements of D, n_p is the number of model parameters (also the dimension of the square matrix D), and the numbering of the model parameters is the same as that used in the output sensitivity matrix $\partial y/\partial \theta$. Because the Cramér-Rao lower bound D is a theoretical lower limit that depends only on data information content and not on any particular estimation method, the merit of an input design for aircraft parameter estimation can be evaluated effectively using D. A common approach is to optimize the input by minimizing the trace of the dispersion matrix, which is the same as minimizing the sum of the Cramér-Rao bounds for the model parameter variances:

$$J\left(u\right) = Tr\left(D\right) \tag{11}$$

Other approaches include minimizing the determinant of D, minimizing the maximum eigenvalue of D, and minimizing a selected parameter covariance from D or some combination of individual D matrix elements.

Computing D requires a prior model of the linear dynamic system, including model parameter values and a discrete measurement noise covariance matrix. Therefore, when optimizing inputs in this way, a complete dynamic system and measurement model is necessary to optimize the input for an experiment intended to collect data for estimating the unknown parameters in a dynamic model. This is the circularity problem discussed earlier.

2.3 Multiple-Input, Multiple-Output Linear Dynamic System with Multiple Parameters

Most of the work in optimal input design for aircraft has been focused on a single-input case wherein the aircraft dynamic motion is in the longitudinal plane of symmetry of the aircraft defined at the start of the maneuver. This longitudinal dynamic motion is excited by a single longitudinal control such as an elevator or stabilator. Studying this single-input case simplifies the optimal input design problem while retaining practical relevance. However, many practical aircraft modeling problems involve multiple inputs. Modern aircraft designs have many controls for enhanced maneuverability, robustness, reduced trim drag, and other reasons. Multiple aircraft controls can excite complex, coupled dynamics and can exhibit interaction effects. Consequently, the multiple-input problem appears often in aircraft stability and control flight testing.

Optimizing multiple inputs involves important considerations that are not present for the single-input case. For example, to quantify the effects of each individual control accurately, the inputs cannot be mutually correlated at a high level, or equivalently, the controls cannot be moved in the same way or in a nearly proportional way. The similarity of the input waveforms can be quantified by the pairwise correlation coefficient, defined as

$$\rho_{12} = \frac{u_1^T u_2}{\sqrt{u_1^T u_1}\sqrt{u_2^T u_2}} \tag{12}$$

where u_1 and u_2 are vectors of discrete input time series for two different controls. The most accurate control effectiveness parameter estimates result from using orthogonal inputs, which are defined as input time series with pairwise correlation coefficient equal to zero. Applying heuristic input designs such as those shown in Fig. 1 for one control at a time is common practice to achieve input orthogonality. However, this time-sequencing approach is inefficient, especially for many controls, and handicaps the excitation of coupled dynamics. The time sequencing approach also precludes estimation of control interaction effects. A superior approach is to use mathematical orthogonality, which can be achieved by designing the input time series with small or zero pairwise correlation magnitudes computed from (12).

Optimized inputs based on a prior model do not implement input orthogonalization directly, but rather design multiple inputs with relatively low correlations in the course of the optimization with respect to a scalar function of the dispersion matrix. Similarly, amplitudes, frequency content, and relative phasing for multiple inputs are determined inherently as part of the multiple-input optimization using a scalar function of the dispersion matrix, while enforcing the practical constraints on the inputs and outputs. Optimizing multiple inputs is also complicated because different aircraft controls have different effectiveness as a function of flight condition, and the input amplitudes interact with the frequency content of the inputs in a complicated way through the aircraft dynamics, which is characterized by a prior model that is inaccurate to an unknown extent.

The multiple-input optimization problem also becomes more difficult computationally as the number of inputs increases, and the difficulty is typically compounded by a corresponding increase in the number of outputs and model parameters. This has been successfully addressed in practice by limiting the possibilities for each of the multiple inputs to only positive, negative, or zero amplitudes, then applying forward dynamic programming to achieve a global optimal solution [3, 14, 20, 22, 24, 32, 37]. The constrained output space is discretized at time stages throughout the maneuver, and input sequences are discarded if they result in either a violated output constraint or a higher cost value to reach any specific discrete output space location at any specific time stage, based on the prior model. The input sequence that reaches a discrete output location within the constraints at the final time with the lowest cost is the global optimal multistep input. This global optimal solution is for the discretized problem with inputs limited to multisteps (or another finite set of input choices), not for the general continuous input optimization problem. The approach is not restricted to linear dynamics for the prior model, but instead can use a prior model with arbitrary nonlinearity and complexity, including actuator dynamics and rate limiting. Full details can be found in [20, 32].

3 Optimal Multiple-Input Design Without a Prior Model

For optimal input design based on a prior model, the inputs are designed to excite the dynamic system response efficiently by concentrating frequency content near the natural frequencies of the dynamic modes, while satisfying practical constraints and optimizing a criterion that is directly related to model parameter estimation. An alternate approach that is very effective and practical is to optimize inputs relative to characteristics of the inputs that are known to provide a robust experiment with informative data for a wide range of possible system dynamics. These characteristics include wideband frequency content, mutually orthogonal inputs, and small dynamic excursions from the desired nominal flight condition. A multiple-input design with these characteristics is the orthogonal optimized multisine input design [25–27, 33, 35], described next.

Orthogonal optimized multisine inputs are multiple perturbation inputs, each consisting of a sum of harmonic sinusoids with unique harmonic frequencies, optimized phase shifts, and specified power distribution. The harmonic frequencies are selected to cover a frequency band of interest for each of the multiple inputs, but the frequency content for each input can be customized. The wideband frequency content of the inputs is important, because there is naturally some uncertainty as to what the modal frequencies of the dynamic system are, and the wideband inputs provide robustness to that uncertainty. Phase shifts for the sinusoidal components of each input are optimized to achieve low peak-to-peak amplitude for the sum of sinusoids used for each input. This input optimization restricts the aircraft responses to small-amplitude variations about the desired flight condition or flight path, which is important for maintaining the validity of the local linear model structure and sometimes also for flight safety or operational constraints. Amplitudes for the individual sinusoidal components that comprise each input can be selected to achieve a specific power distribution with frequency.

The multiple inputs obtained using this design technique are mutually orthogonal in both the time domain and the frequency domain simultaneously, and are designed for high data information content in a short time period, while minimizing excursions from the nominal flight condition or flight path. The mutual orthogonality of the inputs allows simultaneous application of multiple inputs, which reduces the required maneuver time, or equivalently, increases the amount of input energy injected into the dynamic system over a given time period. Even more important for many applications, the inputs provide continuous, effective, multi-axis excitation as the aircraft flies through various flight conditions that may be sustainable for only short periods of time.

Each perturbation input u_j applied to the jth individual control effector is a sum of harmonic sinusoids with individual phase shifts ϕ_{jk},

$$u_j\left(t\right) = \sum_{k\,\in\,\{1,2,\ldots,M\}} A_{jk} \sin\left(\frac{2\pi k\, t}{T} + \phi_{jk}\right) \qquad j = 1,\, 2,\, \ldots,\, n_i \tag{13}$$

where M is the total number of available harmonic frequencies, T is the time length of the excitation, and A_{jk} is the amplitude for the kth sinusoidal component of u_j. Each of the j inputs is the sum of selected components from the pool of M harmonic sinusoids with frequencies $\omega_k = 2\pi k/T, k = 1, 2, \ldots, M$, and $\omega_M = 2\pi M/T$ represents the upper limit of the frequency band for the excitation inputs. The interval $[\omega_1, \omega_M]$ rad/s specifies the range of frequencies where the aircraft dynamics are expected to lie.

The mutual orthogonality of the inputs in the time domain comes from the fact that each input is composed of harmonic sinusoids with the same base period T, but unique harmonic frequencies. Orthogonality in the frequency domain comes from using unique frequencies for the component sinusoids in each multisine input. These mutual orthogonality properties are unaffected by the choice of phase angles for the sinusoidal components of each multisine input [25,26,33].

If the phase angles ϕ_{jk} in (13) were chosen at random on the interval $[0, 2\pi)$ rad, then in general, the various harmonic components would add together at some points to produce a multisine input with relatively large amplitude excursions. This is undesirable, because such inputs can move the aircraft away from the desired flight condition for the maneuver. To prevent this, the phase angles for the selected harmonic components of each multisine input u_j are optimized for minimum relative peak factor RPF, defined as

$$RPF\left(u_j\right) = \frac{\left[\max\left(u_j\right) - \min\left(u_j\right)\right]}{2\sqrt{2}\,\mathrm{rms}\left(u_j\right)} \tag{14}$$

Relative peak factor is a measure of the efficiency of an input for dynamic modeling purposes, equal to the amplitude range of the input divided by a measure of the input energy. Low values of relative peak factor are desirable for highly efficient and effective dynamic modeling, because the objective is to excite the dynamic system with good input energy over a variety of frequencies while minimizing the input amplitude excursions in the time domain, to avoid driving the aircraft too far away from the desired nominal flight condition. For each multisine input u_j, RPF is minimized by adjusting the phase angles for each of the individual sinusoidal components of the multisine input. The resulting optimization problem is nonconvex, as shown in Fig. 3 for a multisine with only two sinusoidal components and therefore two phase angles to choose. A practical problem would have more sinusoidal components, typically 5-20, for which the Nelder-Mead simplex algorithm can be applied to find a solution [25, 26, 33]. Because of the nonconvexity of the problem, the phase angle optimization can be repeated with different random starting values to improve the likelihood of finding a good solution close to the global optimum.

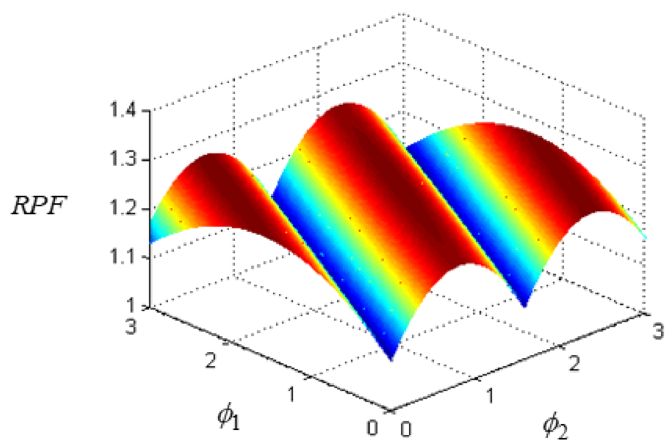

Fig. 3: Nonconvex surface for relative peak factor optimization

After the phase angle optimization for minimum RPF is complete, each orthogonal optimized multisine input is converted to a perturbation input using a one-dimensional search to find a time offset so that the input begins and ends at zero amplitude. This is equivalent to sliding the input along the time axis until a zero crossing occurs at the origin of the time axis. To implement the same time shift for each sinusoidal component of a multisine input, the phase offset for each sinusoidal component must be different, because each has a different frequency. The phase offset $\Delta\phi_{jk}$ to be added to the optimized phase angle ϕ_{jk} for each sinusoidal component is

$$\Delta\phi_{jk} = \omega_{jk}\,\tau \tag{15}$$

where τ is the time shift needed to start the phase-optimized multisine input at zero, and ω_{jk} is frequency k for u_j. Because the sinusoidal components are all harmonics of the base frequency with period T, if the initial value of the input is zero, then the final value of the input at time T will also be zero. The power spectra, relative peak factors, and mutual orthogonality of the inputs are all unaffected by the phase shifts made to produce perturbation inputs [26,33].

The integers k specifying the harmonic frequencies for each multisine input are selected to be unique to that input, but are not necessarily consecutive. A good approach for multiple inputs is to assign consecutive integers to each input alternately. This is illustrated in Fig. 4 for a multiple-input design. There are three inputs in this case, with each input applied to an individual control: elevator, aileron, and rudder. The harmonic frequencies were interleaved among these three inputs to achieve wideband frequency content in each input for robust excitation and to enable accurate individual control surface effectiveness estimates. Because each input has wideband frequency content, the same input design can be applied at various flight conditions, which simplifies the excitation strategy and reduces flight computer memory requirements. Power fraction can be distributed among the sinusoidal components to emphasize frequencies where the system dynamics are more likely to lie, as shown for the elevator and rudder in Fig. 4.

Figure 5 shows inputs designed using the frequency content depicted in Fig. 4 for time length $T = 27$ s. Each multisine input is composed of 14 sinusoids with harmonic frequencies spread evenly over the range [0.1, 1.7] Hz, where the system dynamics are expected to lie. The inputs shown in Fig. 5 can be applied simultaneously, because of the mutual orthogonality of the inputs in both the time and frequency domains.

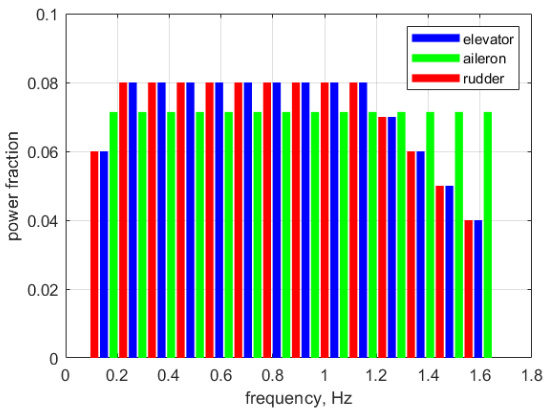

Fig. 4: Multisine input power spectra

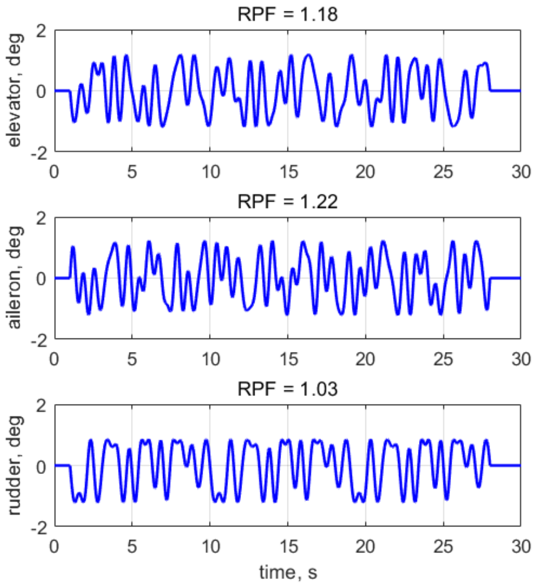

Fig. 5: Orthogonal optimized multisine inputs

This multiple-input design concept has been applied successfully in flight for as many as 16 simultaneous orthogonal optimized multisine inputs [27]. Because the inputs produce small-amplitude aircraft responses by design, it is possible to analyze the resulting flight data using decoupled longitudinal and lateral/directional dynamic equations [33]. This saves flight test time and enhances the dynamic response, because a conventional approach would apply excitation time series to individual controls sequentially, often in separate maneuvers. Because orthogonal optimized inputs can be applied simultaneously, required flight test time is reduced by a factor of the number of controls, resulting in increasingly significant cost savings and efficiency as the number of aircraft controls increases. Furthermore, it is possible to extract control interaction effects from flight test data collected using orthogonal optimized multisine inputs, whereas flight data from sequential application of single inputs does not provide the necessary information for that purpose.

Closer spacing for the harmonic frequencies can be achieved by increasing the time length of the excitation T. For a given excitation frequency band, this increases the number of available harmonic frequencies M that can be distributed among the multiple inputs.

The sinusoidal components in (13) can be assigned arbitrary fractions of the total power in the multisine input, to emphasize the excitation at selected frequencies. This is implemented by choosing component amplitudes as

$$A_{jk} = A_j \sqrt{P_{jk}} \tag{16}$$

where A_j is the amplitude of the multisine input u_j, and P_{jk} is the power fraction for the kth sinusoidal component of u_j. The sum of the power fractions for the sinusoidal components in each multisine input must equal 1,

$$\sum_k P_{jk} = 1 \tag{17}$$

This feature can be used to concentrate input excitation power at frequencies near where the system dynamics are expected to lie, or to avoid frequencies that might excite undesired dynamics such as fuel slosh or structural modes, or to improve dynamic excitation by assigning frequencies according to the capabilities of individual controls or actuators.

In the absence of such knowledge, the best approach is to implement uniform power distribution by computing P_{jk} and A_{jk} as

$$\begin{aligned} P_{jk} &= 1/M_j \qquad \forall k \\ A_{jk} &= A_j \sqrt{P_{jk}} \end{aligned} \tag{18}$$

where M_j is the number of sinusoidal components included in the summation of (13) for u_j, and A_j is the amplitude of the multisine input u_j. For uniform power distribution, selection of the A_{jk} reduces to selecting a single value for the multisine input amplitude A_j. Each multisine input u_j can have arbitrary amplitude A_j, subject to practical flight test and modeling constraints, without altering the power spectra.

Often, the same optimal input design can be used for a wide variety of flight conditions throughout the stability and control flight testing, with only input amplitude adjustments made to account for changing control surface effectiveness and flight conditions. This adjustment is also necessary when there is deficient prior information for the system dynamics, measurement noise, or control surface effectiveness. Although it is possible to obtain information for effective sequential input design in flight from real-time modeling results [23], a more conservative approach is to start with low input amplitudes, then increase the input amplitudes slowly until sufficient signal-to-noise ratios are achieved for the aircraft responses. Signal-to-noise ratios of 10-20 on the aircraft responses are generally sufficient for good modeling results, and the signal-to-noise ratio can be determined readily by rough inspection of the data collected in real time, or by straightforward real-time calculations [33]. Because of the wideband frequency content of each multisine input, only the input amplitudes need to be changed; the orthogonal phase-optimized input waveforms can remain the same.

The inputs are sums of harmonic sinusoids with a common base period T, which means they are periodic for the excitation period T, and the inputs can be applied repeatedly without any discontinuities in magnitude or slope. The input design has minimum RPF with various frequencies and phase angles, which keeps the aircraft response close to the nominal flight condition or flight path and produces responses similar to flight in light to moderate turbulence. The aircraft response also stays near the nominal flight condition or flight path, because each perturbation input is a sum of harmonic sinusoids, which are all balanced about zero (equal enclosed area above and below zero). The result is rich dynamic multi-axis response about the nominal flight condition or flight path.

In practice, pilot inputs and/or automated guidance and feedback control can act to spoil the perfect orthogonality (zero pairwise correlations) of the excitation inputs. However, good modeling results require only low correlations, not zero correlations. The slightly correlated inputs that result from applying orthogonal optimized inputs simultaneously with control movements from the pilot and/or automated guidance and feedback control still result in excellent multi-axis flight data for dynamic modeling.

4 Optimal Multiple-Input Design Case Studies

Optimal multiple-input design will be demonstrated using an F-16 simulation, followed by a presentation of optimal multiple-input designs flown during the X-43A hypersonic stability and control flight tests in 2004.

4.1 F-16 Simulation Case Study

To illustrate the application and effectiveness of optimal multiple-input designs, a lateral/directional linear model of the F-16 aircraft at 5 deg nominal angle of attack and 10,000 ft altitude was used. This model has two controls, aileron and rudder, and was obtained from local linearization of the F-16 nonlinear simulation described in [33] and included in SIDPAC. This linear dynamic model represents a typical prior model for multiple-input design, and was considered the prior model for this case study.

The global optimal multistep input design technique described in [20,32] was applied using the prior model, and the orthogonal optimized multisine input design technique described in [25,26,33,35] was applied without using the prior model. Multiple-input designs using heuristic single inputs, such as those shown in Fig. 1, applied sequentially to one control at a time, were also evaluated. As expected, the sequential heuristic single inputs produced significantly inferior results compared to either of the optimal multiple-input designs, and therefore the results from those inputs are not shown.

To simulate errors in the prior model relative to the actual system dynamics, the global optimal multistep inputs were designed based on the prior model, and the orthogonal optimized multisine inputs were designed using the same maneuver time, frequency range, and input amplitudes. Optimal inputs were designed only once using each method, then the input designs were evaluated using dynamic models with the same linear model structure as the prior model, but with the model parameters varied significantly, randomly, and repeatedly in a Monte Carlo formulation. For each case, the input designs were evaluated by computing the cost function in (11). With this approach, the input designs were evaluated by simulating their application to a wide variety of system dynamics that were different from the prior model. Measurement noise root-mean-square values (i.e., the square roots of the diagonal elements of R) were set as shown in Table 1. These are realistic noise levels for typical flight test instrumentation. All model parameters were varied randomly over a chosen range using a uniform distribution, and selecting a different set of model parameters for each run. Model parameters were varied relative to the prior model parameters over ranges of ± 30 percent for static stability derivatives, ± 40 percent for dynamic stability derivatives, and ± 20 percent for control derivatives. These ranges were determined from flight test experience with prior models used for input design. Prior models are typically obtained from aerodynamic calculations such as computational fluid dynamics (CFD) or from wind tunnel tests. The F-16 nonlinear simulation is based on data from wind tunnel tests and ground-based engine tests [33].

Figure 6 shows the optimal input designs for aileron and rudder for a 10 s maneuver. First-order actuator dynamics were included in the dynamic model, with an actuator time constant of 0.0495 s for both aileron and rudder. Note that any actuator dynamics or limiting are considered part of the prior model, which can be arbitrarily complex for the global optimal multistep input design using forward dynamic programming. Consequently, actuator dynamics and limiting, along with the rest of the prior model, affect the global optimal multistep solution.

Table 1: Measurement noise root-mean-square (RMS) values

Output	RMS noise level
Sideslip angle, β	0.1 deg
Roll rate, p	0.3 deg/s
Yaw rate, r	0.3 deg/s
Roll angle, ϕ	0.05 deg
Lateral acceleration, a_y	0.01 g

Frequency content of the orthogonal optimized multisine inputs is shown in Fig. 7. As is typical in practice, the power spectra were modified from uniform to reflect the higher likelihood that significant system dynamics will lie in the middle of the frequency range for the input design. The global optimal multistep design technique had information of this kind from the prior model, although the prior model was inaccurate in various ways and to varying extents for each Monte Carlo run. The maximum amplitude for both inputs was set at 1 deg, which kept the inputs within ± 1 deg. The minimum pulse widths for the global optimal multistep inputs were set to 0.5 s, so that the frequency range for both input designs was approximately the same. The multistep inputs had an advantage, because the sharp corners of a multistep input produce additional frequency content, due to the need for many sinusoids with various frequencies to represent sharp corners in the time domain. This additional frequency content improves robustness to errors in the prior model. Output amplitude constraints used in the global optimal multistep input design were ± 1 deg for sideslip angle β and ± 10 deg for roll angle ϕ. The orthogonal optimized multisine inputs with the same amplitudes also satisfied these output constraints, based on the response calculated with the prior model.

Figure 8 shows cost function values obtained using the global optimal multistep input design and the orthogonal optimized multisine input design for 500 Monte Carlo runs. Performance was excellent in both cases, with the global optimal multistep input achieving slightly superior results, indicated by more Monte Carlo runs with lower cost function values. However, the global optimal multistep input violated the output constraints more often, where a constraint violation was defined as any Monte Carlo run where at least one output constraint was violated by more than 20 percent. For the global optimal multistep input, output constraints were violated in 163 of the 500 Monte Carlo runs, whereas the orthogonal optimized multisine input had 59 output constraint violations. This demonstrates improved robustness

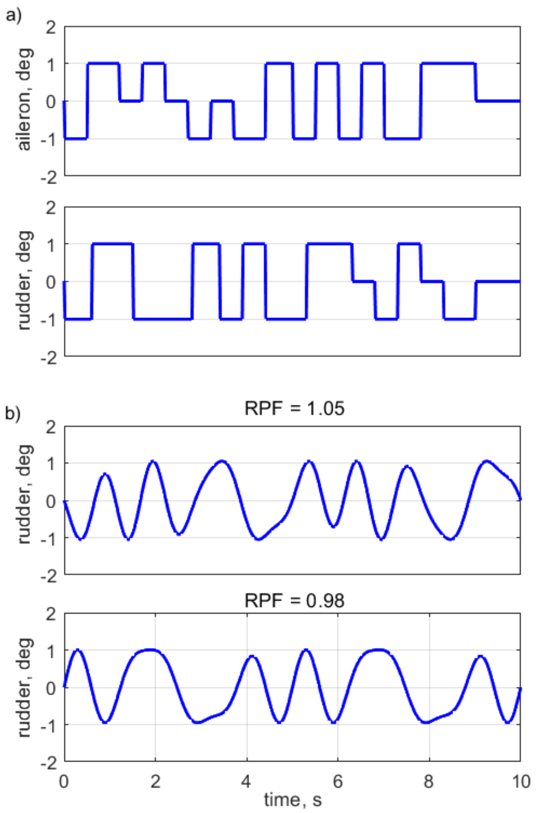

Fig. 6: Optimal inputs: a) global optimal multistep, b) optimized multisine

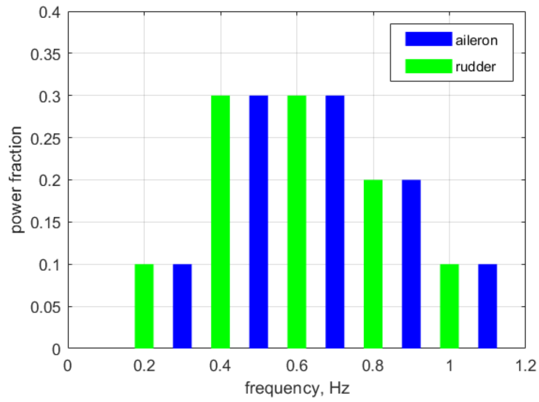

Fig. 7: Optimized multisine input power spectra

for the orthogonal optimized multisine input design, which is related to the lower input energy for this design, compared to the global optimal multistep input. Total input energy (integrated absolute values) of the global optimal multistep inputs was 15.5, compared to 12.8 for the orthogonal optimized multisine inputs. The global optimal multistep input design technique uses the information in the prior model to minimize the cost function related to model parameter uncertainties, but in doing so, can sometimes violate the output amplitude constraints, because the prior model is not an accurate representation of the real system dynamics. Note also that the global optimal multistep input was designed specifically to minimize the cost function in (11) used to create Fig. 8, whereas the orthogonal optimized multisine input was designed and optimized in a completely different way, based on input characteristics.

The global optimal multistep input design technique did not produce mutually orthogonal inputs, because the inputs were designed to minimize the cost function in (11), based on the prior model. However, these inputs still had very low correlation. The pairwise correlation coefficient computed from (12) for the global optimal multistep inputs was -0.19, whereas the pairwise correlation coefficient for the orthogonal optimized multisine inputs was 0.00, by design.

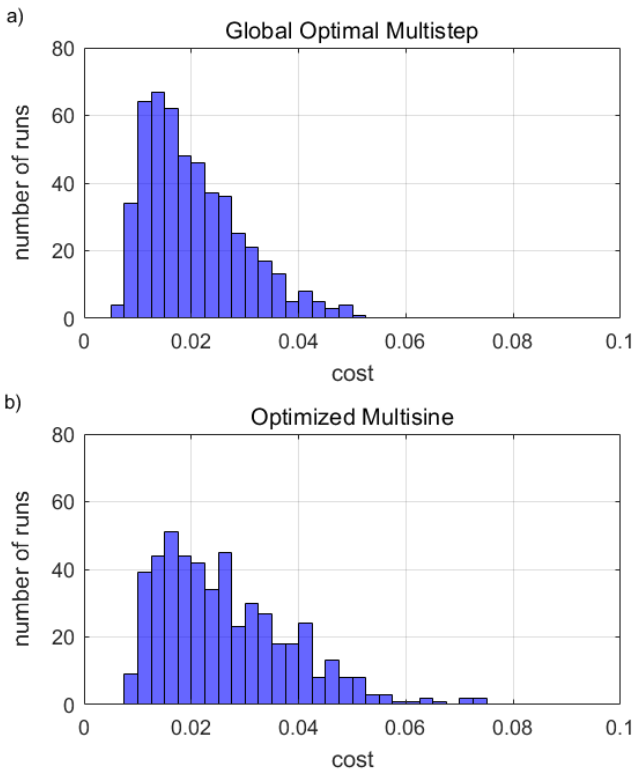

Fig. 8: Monte Carlo results: a) global optimal multistep, b) optimized multisine

The measurement noise levels and the magnitudes of the random errors used to generate the actual system dynamics for the Monte Carlo results shown in Fig. 8 are representative of a real practical flight test input design situation. As the differences between the prior model and the actual system dynamics were made even greater, the histogram for the global optimal multistep input became more similar to the histogram associated with the orthogonal optimized multisine input. Both optimal multiple-input design techniques exhibited excellent robustness to parameter errors in the prior model.

Because the global optimal multistep input design involves a forward dynamic programming solution based on time integrations using the prior model, the computing time and effort were higher than for the orthogonal optimized multisine input design. For this case study, the global optimal multistep input design required 235.4 s on an HP EliteBook 840 G6 Windows 10 laptop computer with background tasks turned off, whereas the orthogonal optimized multisine input design required 1.8 s under the same conditions. This excludes the time and effort required to generate the prior model required for the global optimal multistep input design. Note that any modern computer with similar performance would work as well – this particular computer was specified only to provide information on the computing hardware used to produce the stated timing results.

The global optimal multistep input design involves design choices such as the output amplitude constraints, the discretization for the constrained output space and the time stages, and the restricted set of possible inputs. The cost function in (11) depends nonlinearly on these design choices. Furthermore, the information for some model parameters (e.g., damping parameters) improves with free response data. These are the reasons that zero amplitudes appear in the global optimal multistep inputs. Note also that if there were a better input sequence, subject to the design choices, then the forward dynamic programming would have found it, because the optimization approach is an efficient exhaustive enumeration based on Bellman's principle of optimality. The global optimal multistep solution is a global optimum subject to the specified design choices and the prior model, and therefore is not the global optimal solution for the continuous problem. However, the global optimal multistep input for the discretized problem produces excellent results in practice, with a reasonable computational cost.

The RPF optimization criteria used to design orthogonal optimized multisine inputs is not directly related to the modeling problem, but instead is a characteristic of the input. However, orthogonal multisine inputs with minimized RPF produce excellent multi-axis data information content and low cost values from (11) while addressing the practical problems of flight test efficiency, avoiding large variations in flight condition or exciting undesired complex effects, avoiding the circularity problem associated with prior models, and enabling flight testing in a wide variety of flight test situations. Furthermore, the inputs have wideband frequency content, which is known to be important for robust and effective excitation inputs. Consequently, minimizing the RPF for orthogonal multisine inputs is a good, practical (albeit indirect) criteria to use for optimal multiple-input design.

Figure 9 shows a cross plot of the two optimal input designs. For the global optimal multistep inputs, the cross plot in Fig. 9a looks the same as a face-centered central composite experiment design often used in static design of experiments [19], because the input possibilities were limited to positive, negative, and zero amplitudes only. Note that this optimal input design approach can use other input possibilities besides multisteps, but multisteps were chosen because of the spectral spread from the corners, high input energy for given maximum input amplitude, and ease of implementation. The cross plot for the orthogonal optimized multisine inputs in Fig. 9b shows that this input design provides richer information content for control interaction effects, because of the many combinations of input amplitudes realized throughout the maneuver.

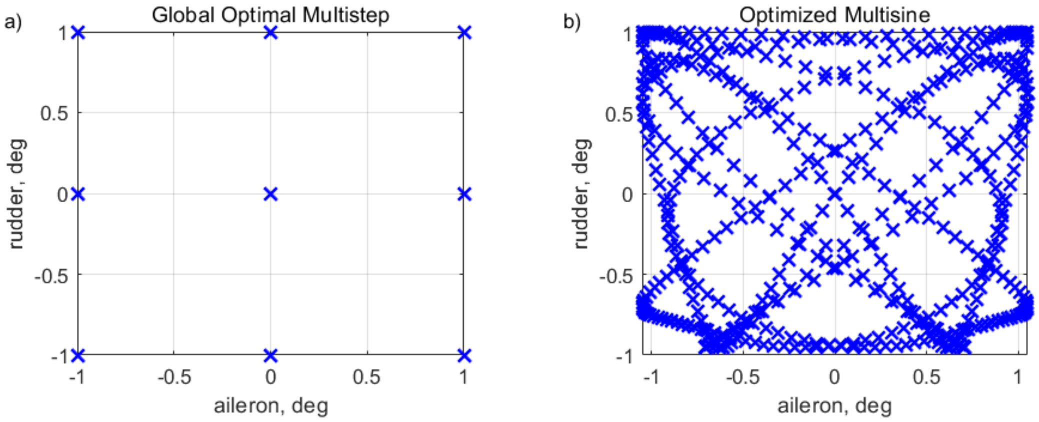

Fig. 9: Input cross plots: a) global optimal multistep, b) optimized multisine

Although a test pilot can approximate either global optimal multistep inputs [21] or orthogonal optimized multisine inputs [1], the practical benefits of automated excitation inputs are well worth the cost and complexity required to build that capability into the flight control system. Automated excitation input capability improves the input implementation accuracy, particularly for multiple inputs, and allows the test pilot or guidance system to fly the aircraft to the desired flight test condition, then activate the automated excitations to efficiently generate informative multi-axis flight data. This reduces pilot workload, improves the dynamic excitation, and enables stability and control flight testing in flight conditions that may be difficult to achieve or sustain.

4.2 X-43A Hypersonic Flight Test Case Study

Both the global optimal multistep and orthogonal optimized multisine multiple-input design techniques were applied for the hypersonic stability and control flight tests conducted in 2004 on the X-43A (Hyper-X) vehicle, shown in Fig. 10a. Figure 10b shows the mission profile for the X-43A flight test, which includes a climb to altitude aboard a B-52 aircraft and a boost phase to the desired hypersonic flight test condition for the scramjet engine test using a Pegasus booster. After the X-43A scramjet engine test was complete, the engine was shut down and a multiple-input global optimal multistep was applied for stability and control flight testing with the engine cowl door open. This input design was chosen because the maneuver was done at a single flight condition, with a desire for the highest efficiency and modeling accuracy possible. The resulting flight test data and modeling results are classified and therefore cannot be shown, but the maneuver execution and modeling were very successful and achieved the research goals.

After that maneuver, the engine cowl door was closed, and a series of orthogonal optimized multisine input maneuvers were conducted at various Mach numbers as the vehicle descended in hypersonic glide. The multiple-input design flown during the descent is shown in Figs. 4 and 5, described earlier. Example modeling results are shown in Fig. 11 for nondimensional aerodynamic roll, pitch, and yaw moments at Mach 5. The nondimensional moment coefficient data shown in Fig. 11 were computed from flight test measurements and the nonlinear equations of motion. This data exhibited low noise levels, because flight measurement noise levels were very low for hypersonic flight in the smooth air of the upper atmosphere. Multiple-input design was critical for the descent maneuvers, because flight conditions changed rapidly during the descent, so that a maneuver using heuristic inputs applied sequentially to individual controls would take too long, and therefore could not accomplish the goal of determining a full set of stability and control characteristics for the vehicle at specific Mach number flight conditions. Furthermore, hypersonic flight test time is expensive and rare, so that an optimal multiple-input design was essential for extracting the most information possible from the stability and control flight testing. For the descent maneuvers, the robustness of the orthogonal optimized multisine input design was desired, so that the same input design could be applied throughout the descent, with changes made only to the individual input amplitudes to account for different individual control effectiveness and changes in dynamic pressure as the vehicle descended into the atmosphere with rapidly changing Mach number. This approach simplified flight test operations and saved memory in the flight control computer.

Figure 11 shows that the nondimensional aerodynamic moment modeling results were excellent in all axes for the maneuver at Mach 5. Similar modeling results were obtained at other Mach numbers, and modeling results were excellent for the nondimensional aerodynamic forces as well [26,30]. Although the flight data from the descent maneuvers are unclassified, the modeling results are access-restricted and cannot be shown. Modeling results based on flight test data obtained by executing orthogonal optimized multisine inputs at hypersonic speeds during the descent were very accurate and achieved project goals. For both of the X-43A flight tests (the first with maximum Mach 7, the second with maximum Mach 10), optimized multiple-input designs were applied to achieve efficient dynamic excitation and accurate modeling of the nondimensional aerodynamics at hypersonic speeds in all rigid-body degrees of freedom simultaneously. Similar success was achieved with orthogonal optimized multisine inputs flown during hypersonic stability and control flight testing for the X-51A Waverider vehicle [34].

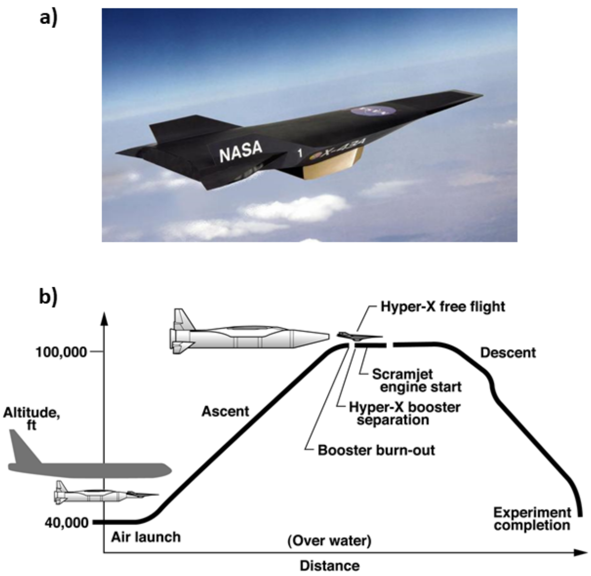

Fig. 10: X-43A Hyper-X: a) vehicle, b) flight test (credit: NASA)

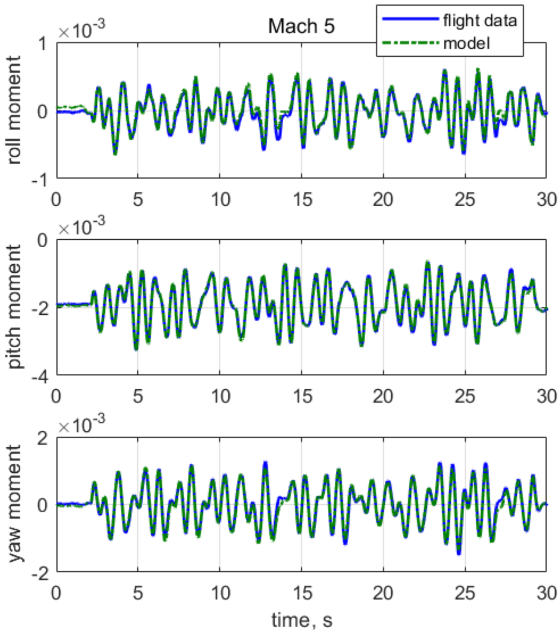

Fig. 11: X-43A Mach 5 modeling results

5 Conclusions

Theoretical and practical aspects of optimal input design for aircraft stability and control flight testing were explained and demonstrated. The flight test input optimization problem formulation and solution methods were discussed. The distinction between optimizing inputs based on a prior model and optimizing inputs with respect to characteristics of the inputs was explored. Optimal multiple-input design case studies were presented for a realistic F-16 simulation and for the X-43A (Hyper-X) hypersonic flight test.

The global optimal multistep and orthogonal optimized multisine input designs have been shown to be effective and practical, both in realistic simulation and in real flight-test applications. The global optimal multistep input design offers the best performance, but requires a prior model, shows reduced robustness to violating output constraints, and has relatively high computational cost that increases with the number of inputs and outputs, and with finer discretizations of time and the constrained output space. The orthogonal optimized multisine input exhibits excellent performance with good robustness to violating output constraints and rich information content for control interaction effects, at a much lower computational cost and without the need for a prior model. Both optimal multiple-input design methods produce inputs with excellent efficiency, effectiveness, and robustness to deficiencies in prior information.

Future flight testing will involve new aircraft designs with multiple controls, complex interaction effects, and possibly inaccurate or poorly-defined prior models. For example, urban air mobility aircraft designs employ both propulsion and aerodynamic controls, with significant complex interaction effects that are not easily characterized by wind-tunnel tests or computational aerodynamics. Many practical flight test situations require rapid and effective interrogation of the aircraft stability and control characteristics using low-amplitude excitations applied to multiple inputs. These considerations, combined with the high cost of flight testing, highlight the value and importance of robust optimized multiple-input designs for aircraft stability and control flight testing.

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